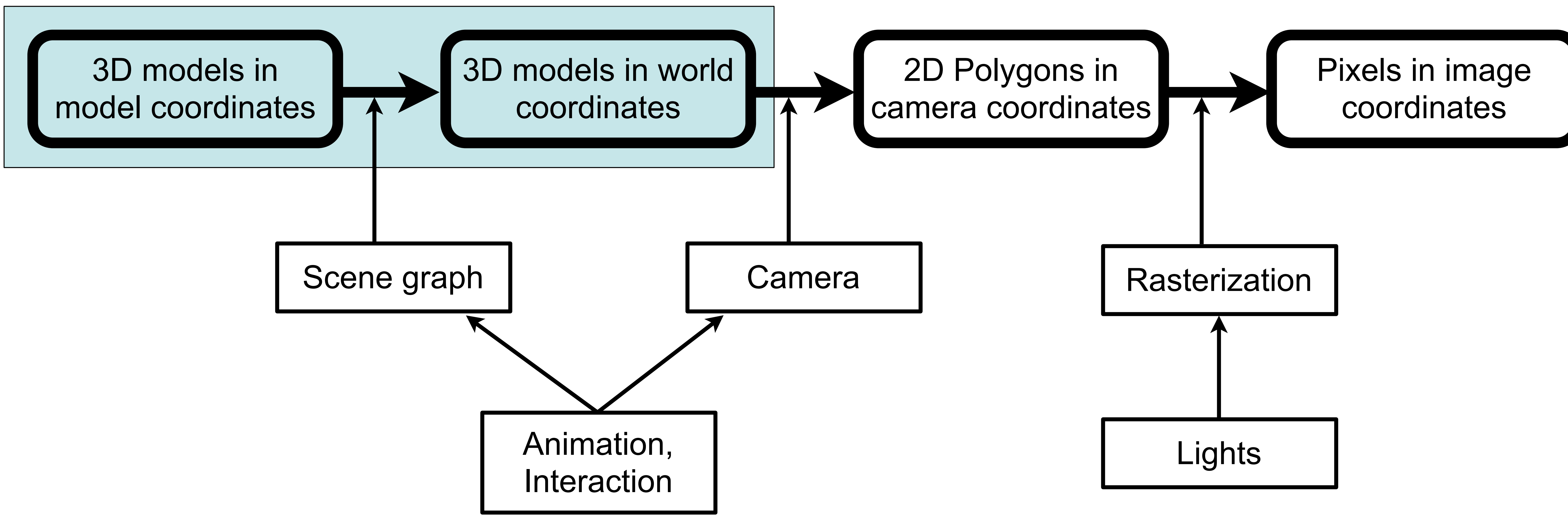
Chapter 3 - 3D Modeling

- Polygon Meshes
- Geometric Primitives
- Interpolation Curves
- Levels Of Detail (LOD)
- Constructive Solid Geometry (CSG)
- Extrusion & Rotation
- Volume- and Point-based Graphics

The 3D rendering pipeline (our version for this class)

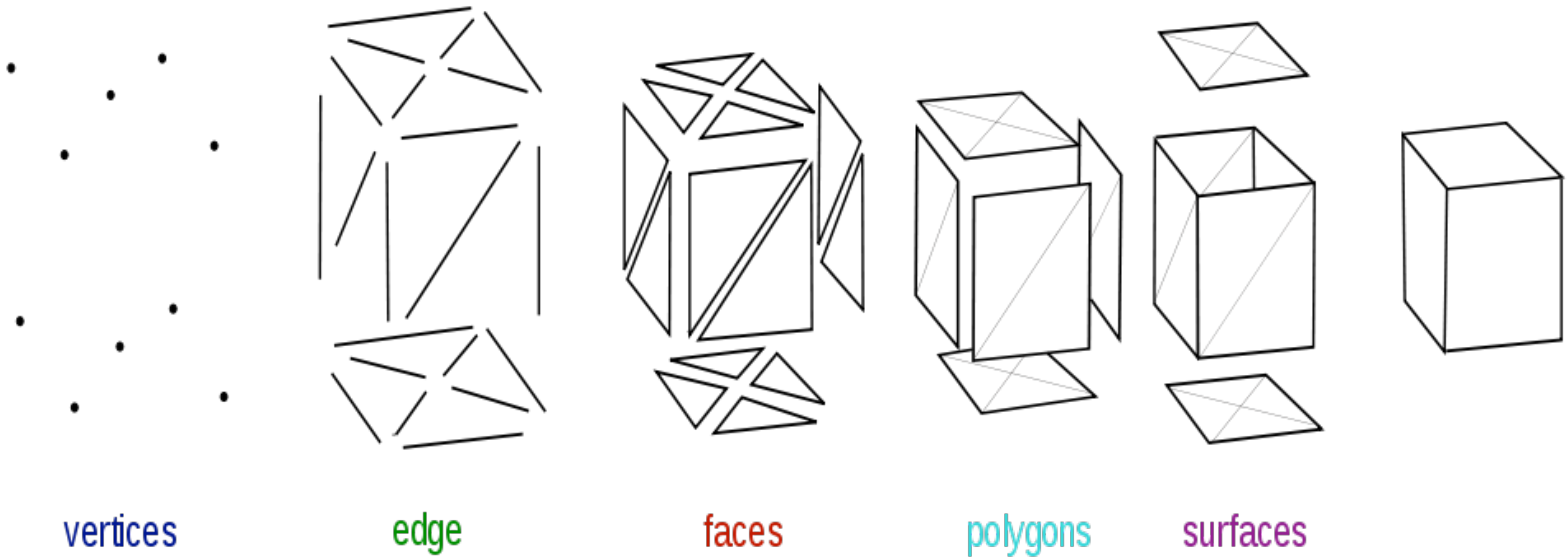


Representations of (Solid) 3D Objects

- Complex 3D objects need to be constructed from a set of primitives
  - Representation schema is a mapping of 3D objects --> primitives
  - Primitives should be efficiently supported by graphics hardware
- Desirable properties of representation schemata:
  - Representative power: Can represent many (or all) possible 3D objects
  - Representation is a mapping: Unique representation for any 3D object
  - Representation mapping is injective: Represented 3D object is unique
  - Representation mapping is surjective: Each possible representation value is valid
  - Representation is precise, does not make use of approximations
  - Representation is compact in terms of storage space
  - Representation enables simple algorithms for manipulation and rendering
- Most popular on modern graphics hardware:
  - *Boundary representations* (B-Reps) using *vertices*, *edges* and *faces*.

Polygon Meshes

- Describe the surface of an object as a set of polygons
- Mostly use triangles, since they are trivially convex and flat
- Current graphics hardware is optimized for triangle meshes



http://en.wikipedia.org/wiki/File:Mesh_overview.svg

3D Polygons and Planes

- A polygon in 3D space should be *flat*, i.e. all vertices in one 2D plane
  - Trivially fulfilled for triangles
- Mathematical descriptions of a 2D plane in 3D space (hyperplane)
  - Method 1: Point *p* and two non-parallel vectors *v* and *w*

$$x = p + s\vec{v} + t\vec{w}$$

  - Method 2: Three non-collinear points
    (take one point and the difference vectors to the other two)
  - Method 3: Point *p* and normal vector *n* for the plane

$$\vec{n} \cdot (x - \vec{p}) = 0$$ using the dot product

  - Method 4: Single plane equation

$$Ax_1 + Bx_2 + Cx_3 + D = 0$$ *A*, *B*, *C*, *D* real numbers

(*A*, *B*, *C*) is the normal vector of the plane

- All description methods easily convertible from one to the other

(E.g. using cross product to compute normal vector)

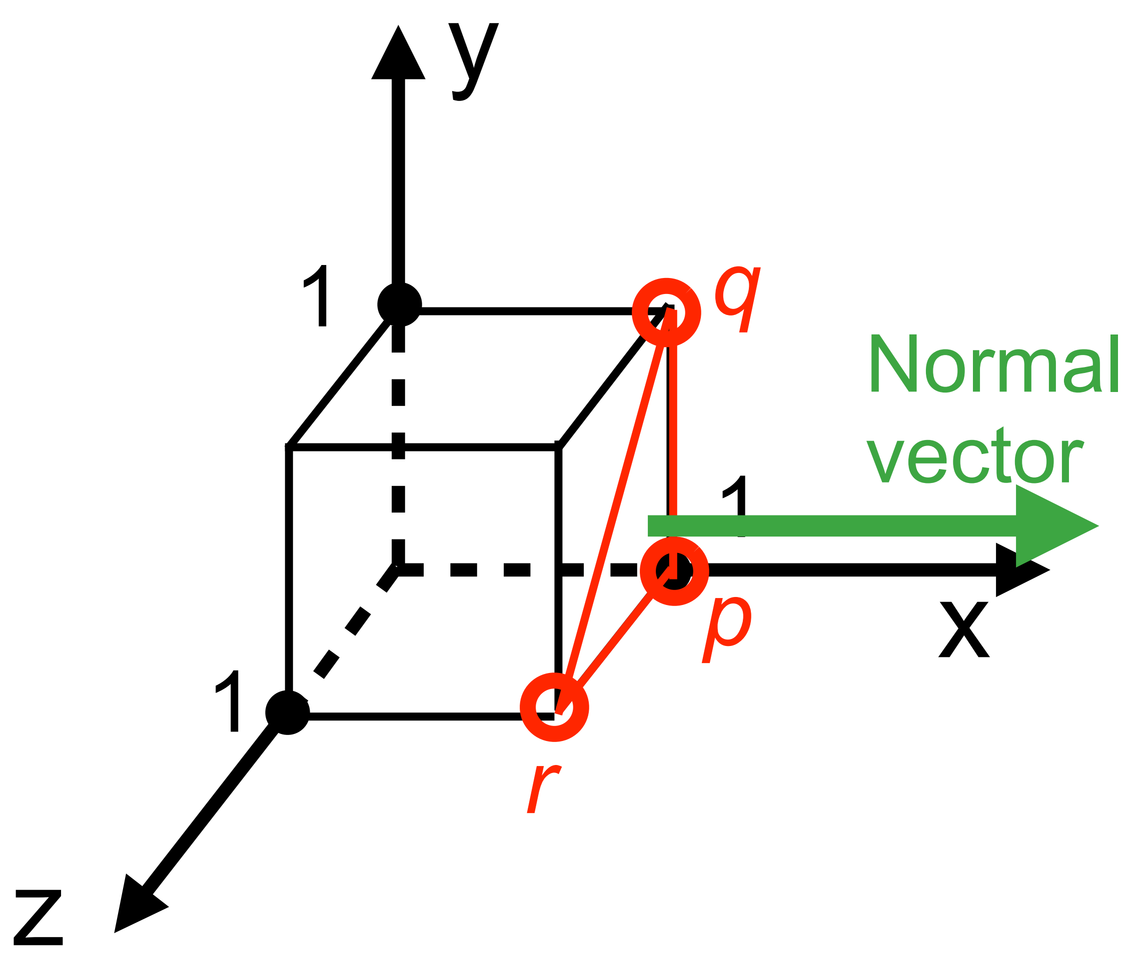
Example: Triangle and Associated Plane

Three points (corners of the unit cube)

$$p = (1,0,0)$$

$$q = (1,1,0)$$

$$r = (1,0,1)$$



Two in-plane vectors:

$$\vec{v} = q - p = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \vec{w} = r - p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Normal vector:

$$\vec{v} \times \vec{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix} = \begin{pmatrix} 1-0 \\ 0-0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Plane equation:

$$1x_1 + 0x_2 + 0x_3 + D = 0$$

Inserting any point gives:

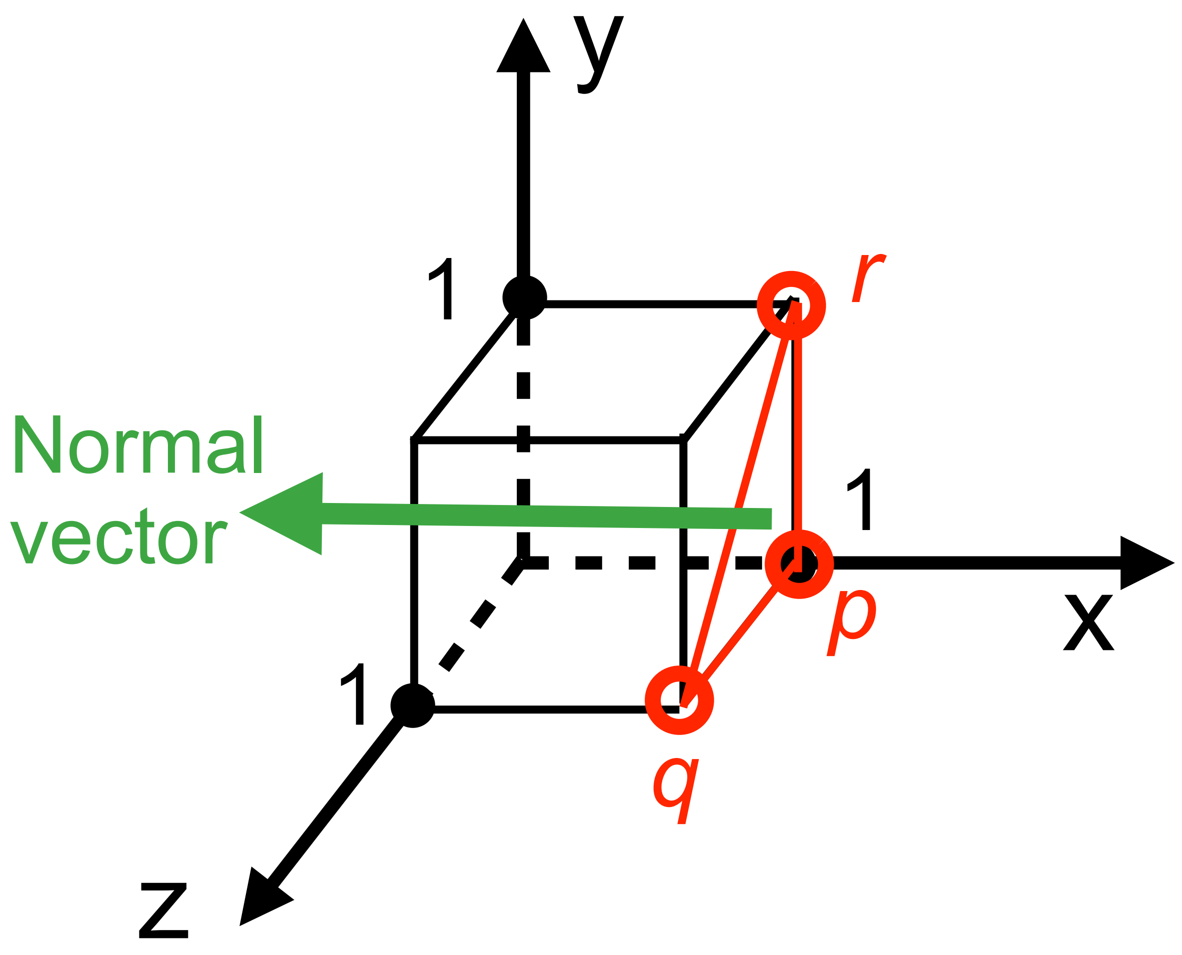
$$x_1 - 1 = 0$$

Example (contd.): Triangle Front and Back Face

Three points (corners of the unit cube)

$$p = (1,0,0)$$
$$q = (1,0,1)$$
$$r = (1,1,0)$$

Different order of vertices: mathematically negative (clockwise)



Normal vector: $\vec{v} \times \vec{w} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

Plane equation:

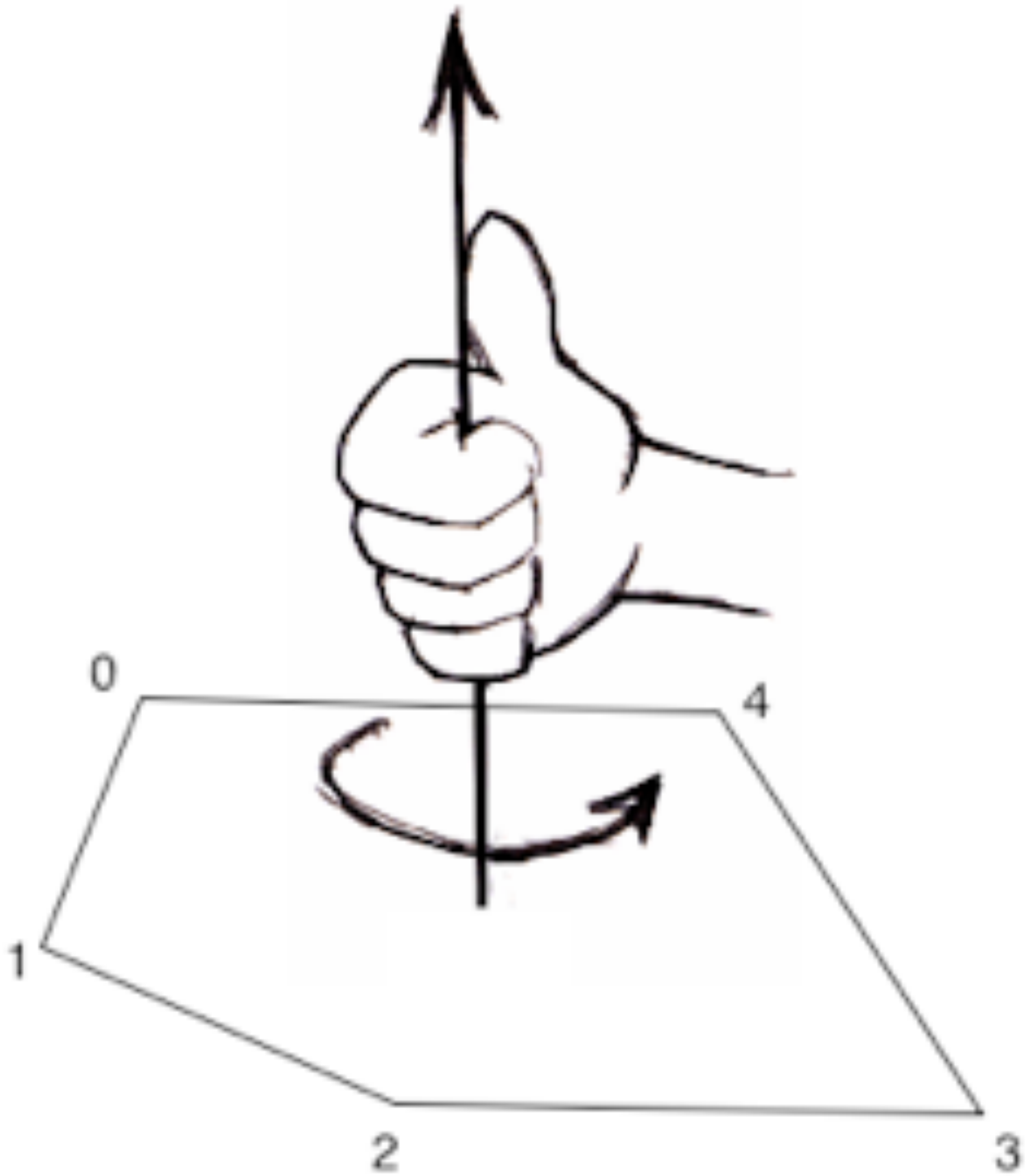
$$(-1)x_1 + 0x_2 + 0x_3 + D = 0$$

Inserting any point gives:

$$-x_1 + 1 = 0$$

For an arbitrary point:
Left hand side of plane equation gives value > 0: Point is in front of plane
Left hand side of plane equation gives value < 0: Point is behind plane

Right Hand Rule for Polygons



- A “rule of thumb” to determine the front side
(= direction of the normal vector) for a polygon
- Please note: The relationship between vertex order and normal vector is just a *convention*!
  - Q: How can we see this from the previous slides?

Source: http://www.csse.monash.edu.au/~cema

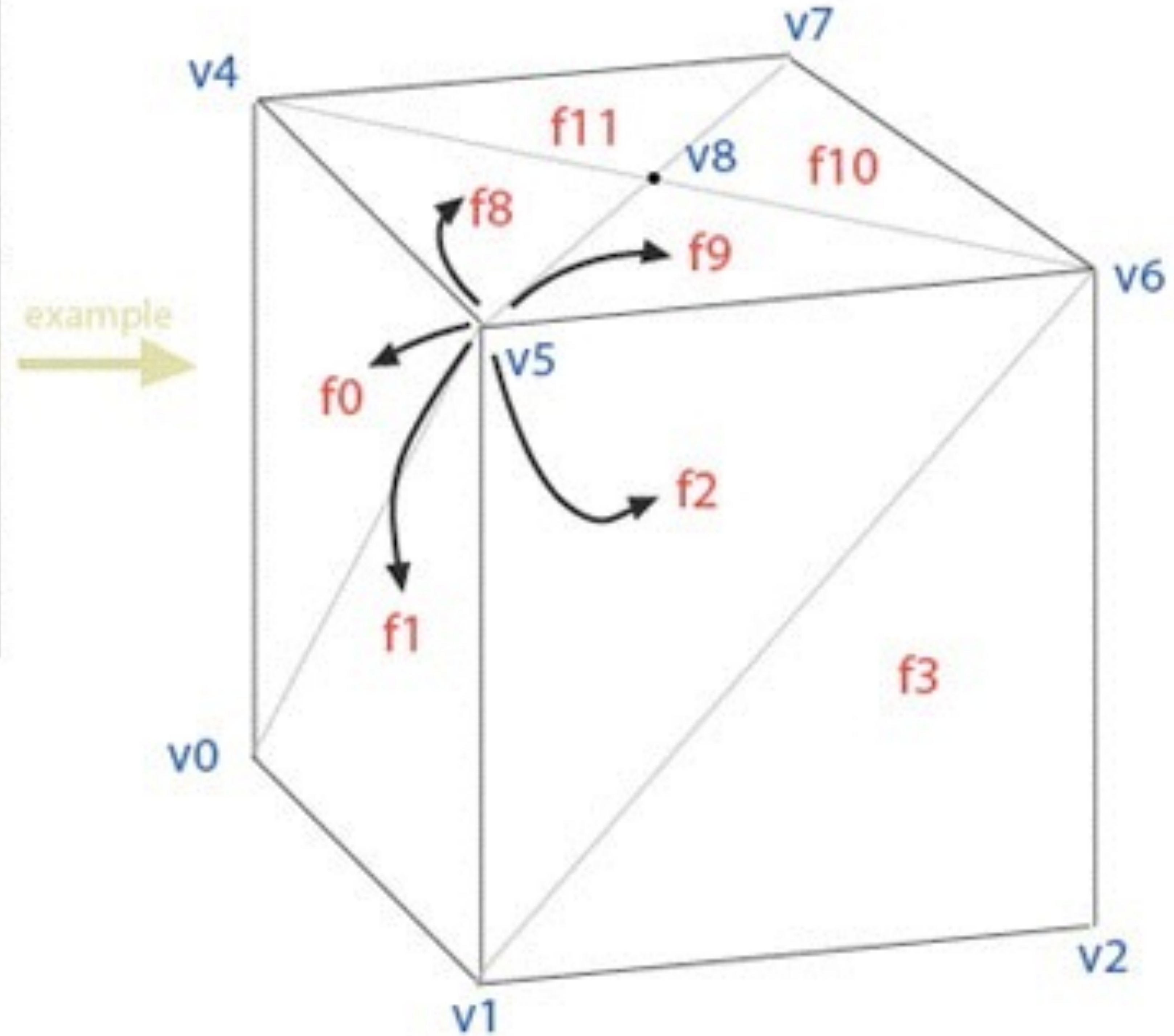
Face-Vertex Meshes

Face List

f0	v0 v4 v5
f1	v0 v5 v1
f2	v1 v5 v6
f3	v1 v6 v2
f4	v2 v6 v7
f5	v2 v7 v3
f6	v3 v7 v4
f7	v3 v4 v0
f8	v8 v5 v4
f9	v8 v6 v5
f10	v8 v7 v6
f11	v8 v4 v7
f12	v9 v5 v4
f13	v9 v6 v5
f14	v9 v7 v6
f15	v9 v4 v7

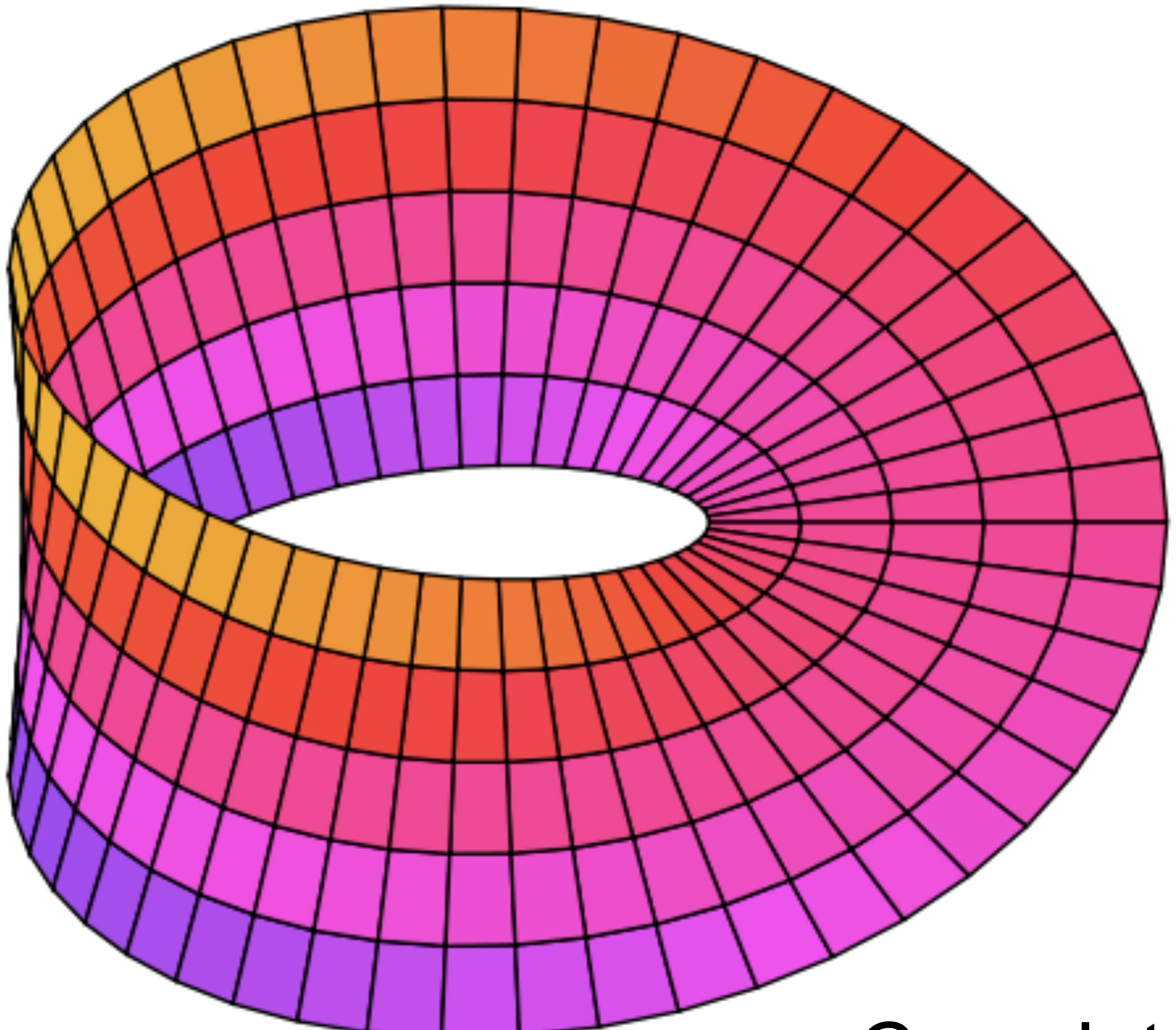
Vertex List

v0	0,0,0	f0 f1 f12 f15 f7
v1	1,0,0	f2 f3 f13 f12 f1
v2	1,1,0	f4 f5 f14 f13 f3
v3	0,1,0	f6 f7 f15 f14 f5
v4	0,0,1	f6 f7 f0 f8 f11
v5	1,0,1	f0 f1 f2 f9 f8
v6	1,1,1	f2 f3 f4 f10 f9
v7	0,1,1	f4 f5 f6 f11 f10
v8	.5,.5,0	f8 f9 f10 f11
v9	.5,.5,1	f12 13 14 15

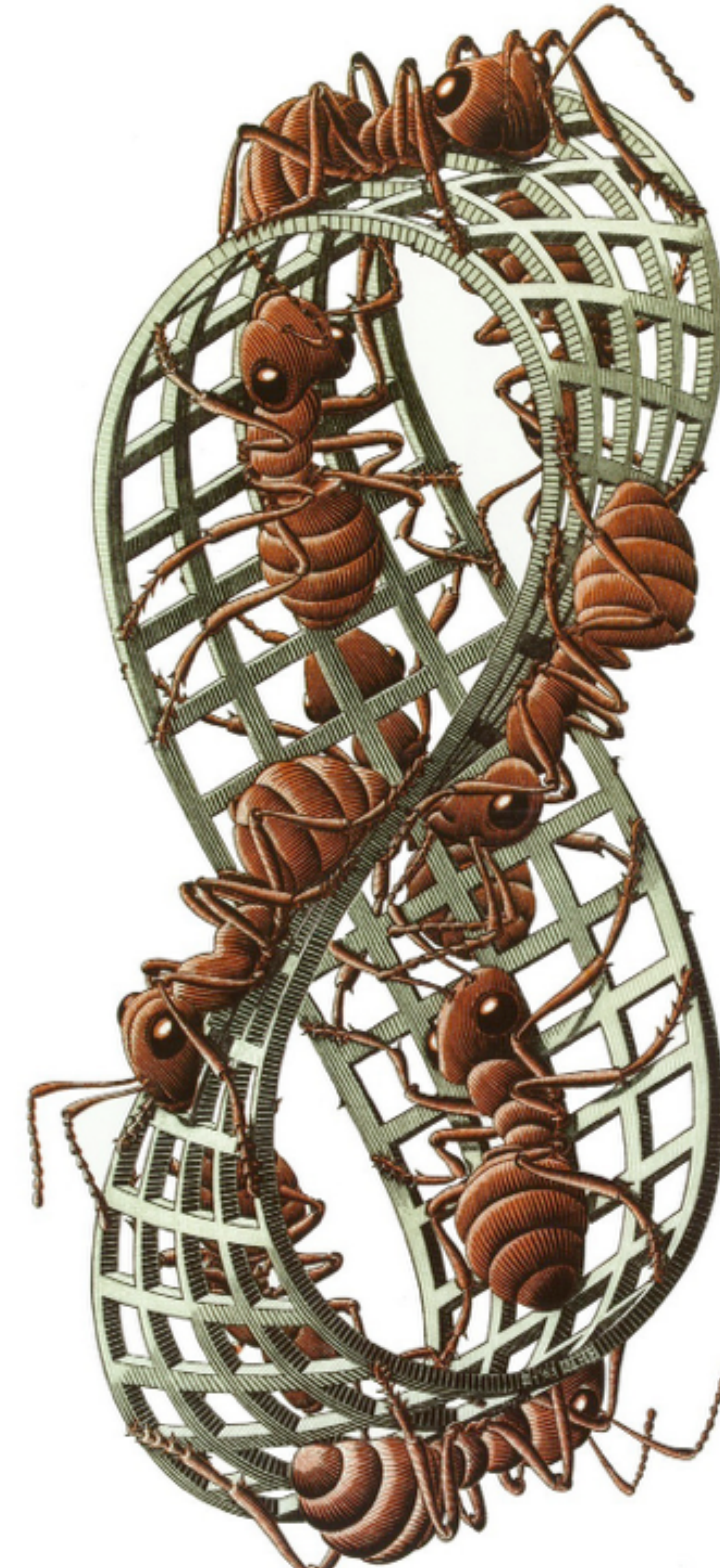


http://en.wikipedia.org/wiki/File:Mesh_fv.jpg

Möbius Strip: Non-Orientable Surface



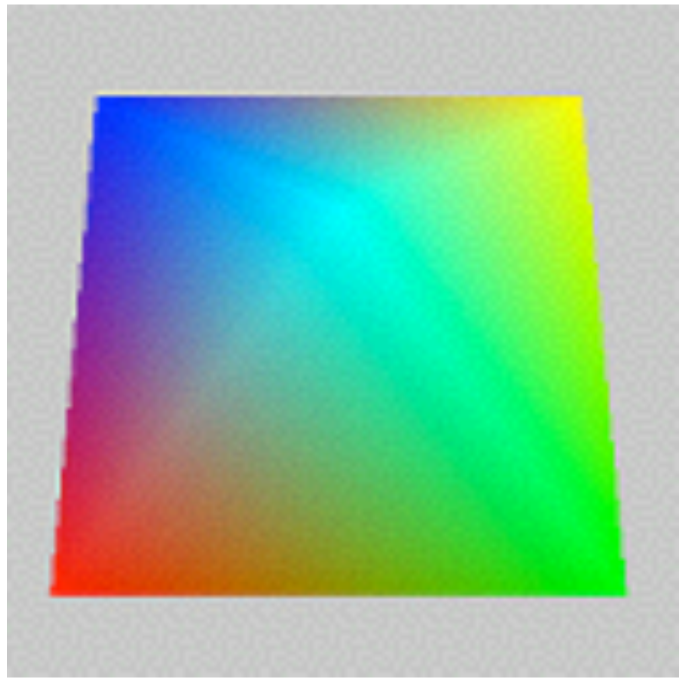
Complete object:
Does not have a
front and back side!



M. C. Escher: Moebius Strip II

Polygon Meshes: Optional data

- Color per vertex or per face: produces colored models
- Normal per face:
  - Easy access to front/back information (for visibility tests)
- Normal per vertex:
  - Standard computation accelerated (average of face normals)
  - Allows free control over the normals
    - use weighted averages of normals
    - mix smooth and sharp edges (VRML/X3D: crease angles)
    - wait for shading chapter ;-)
- Texture coordinates per vertex
  - wait for texture chapter ;-)



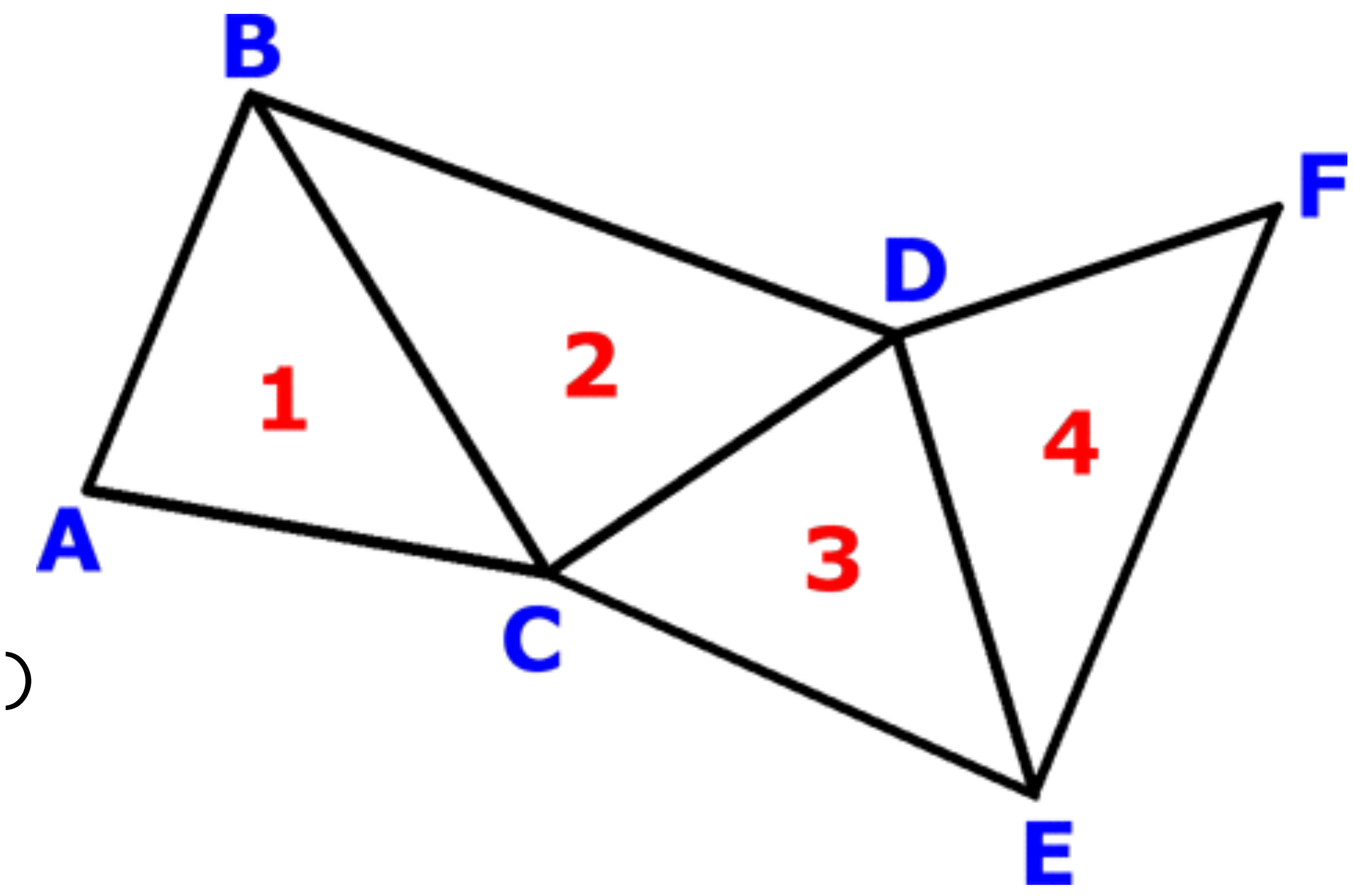
http://en.wikipedia.org/wiki/File:Triangle_Strip.png

Polygon Meshes: other descriptions

- Other representations for polygon meshes exist
  - optimized for analyzing and modifying topology
  - optimized for accessing large models
  - optimized for fast rendering algorithms
  - optimized for graphics hardware

- Example: triangle strip
  - needs N+2 points for N polygons
  - implicit definition of the triangles
  - optimized on graphics hardware
  - OpenGL / JOGL:

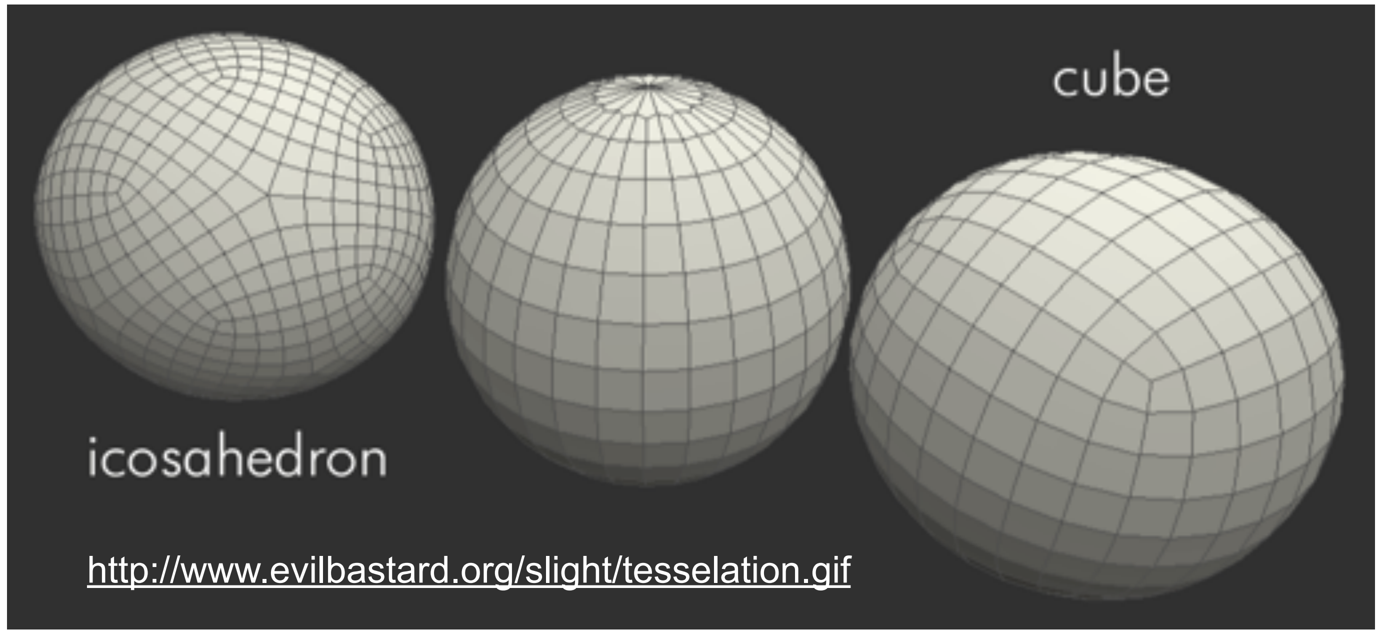
```
gl.glBegin(GL2.GL_TRIANGLE_STRIP)
     gl.glVertex3d(-1, -1, 1);
     ...
```

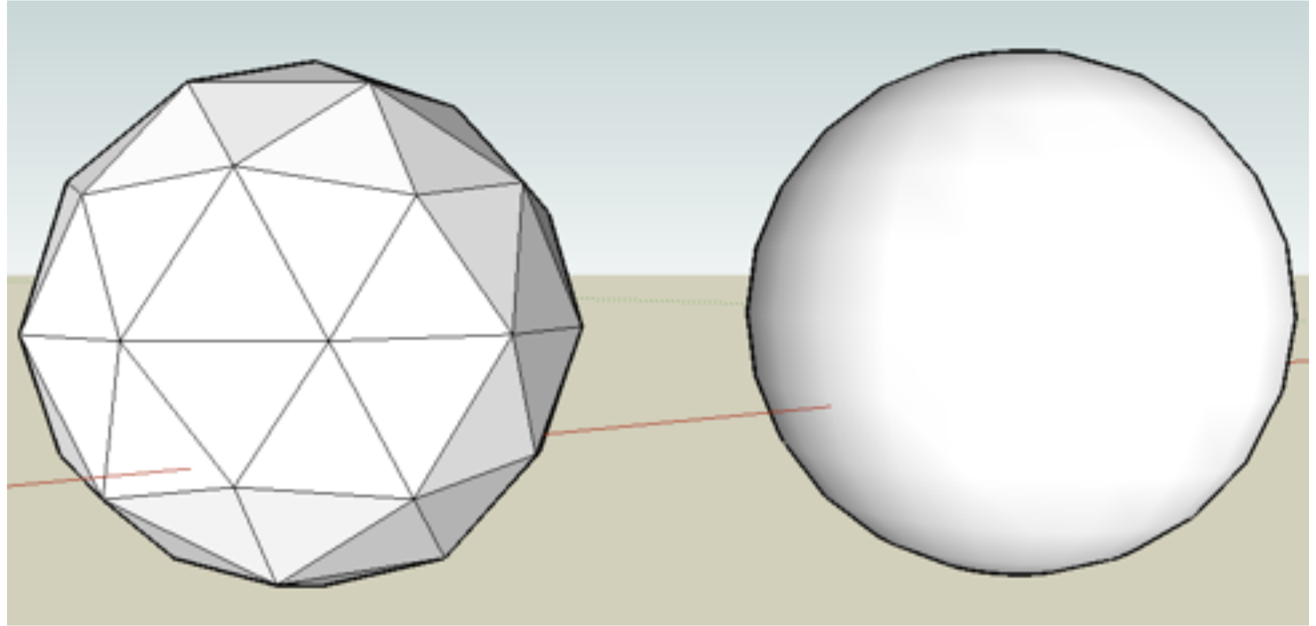


http://en.wikipedia.org/wiki/File:Triangle_Strip.png

Approximating Primitives by Polygon Meshes

- Trivial for non-curved primitives...
- The curved surface of a cylinder, sphere etc. must be represented by polygons somehow (Tesselation).
- Not trivial, only an approximation and certainly not unique!
  - GLU utility functions for tesselation exist
- Goal: small polygons for strong curvature, larger ones for areas of weak curvature
  - This means ideally constant polygon size for a sphere
  - Where do we know this problem from??? Something playful...





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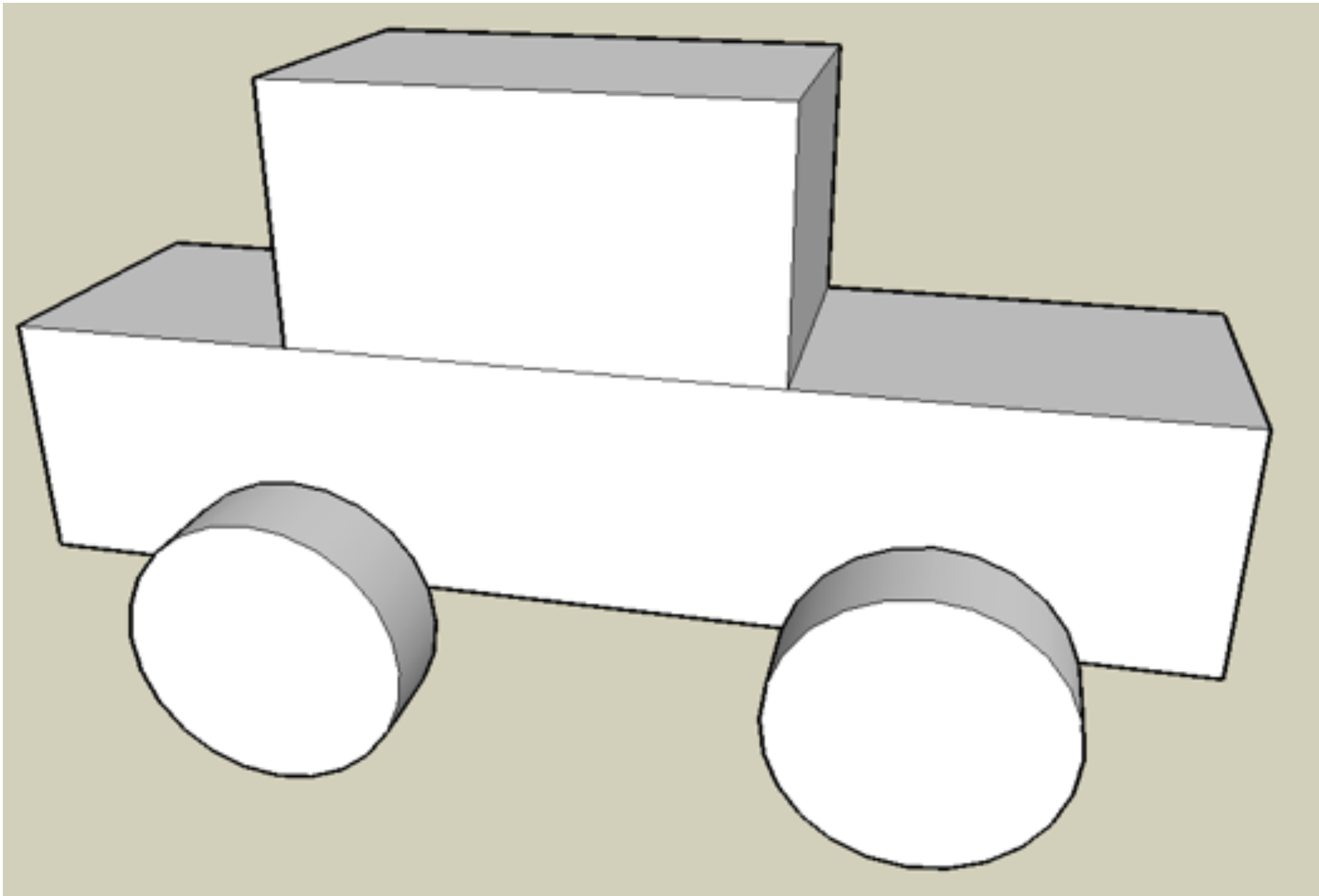
Geometric Primitives

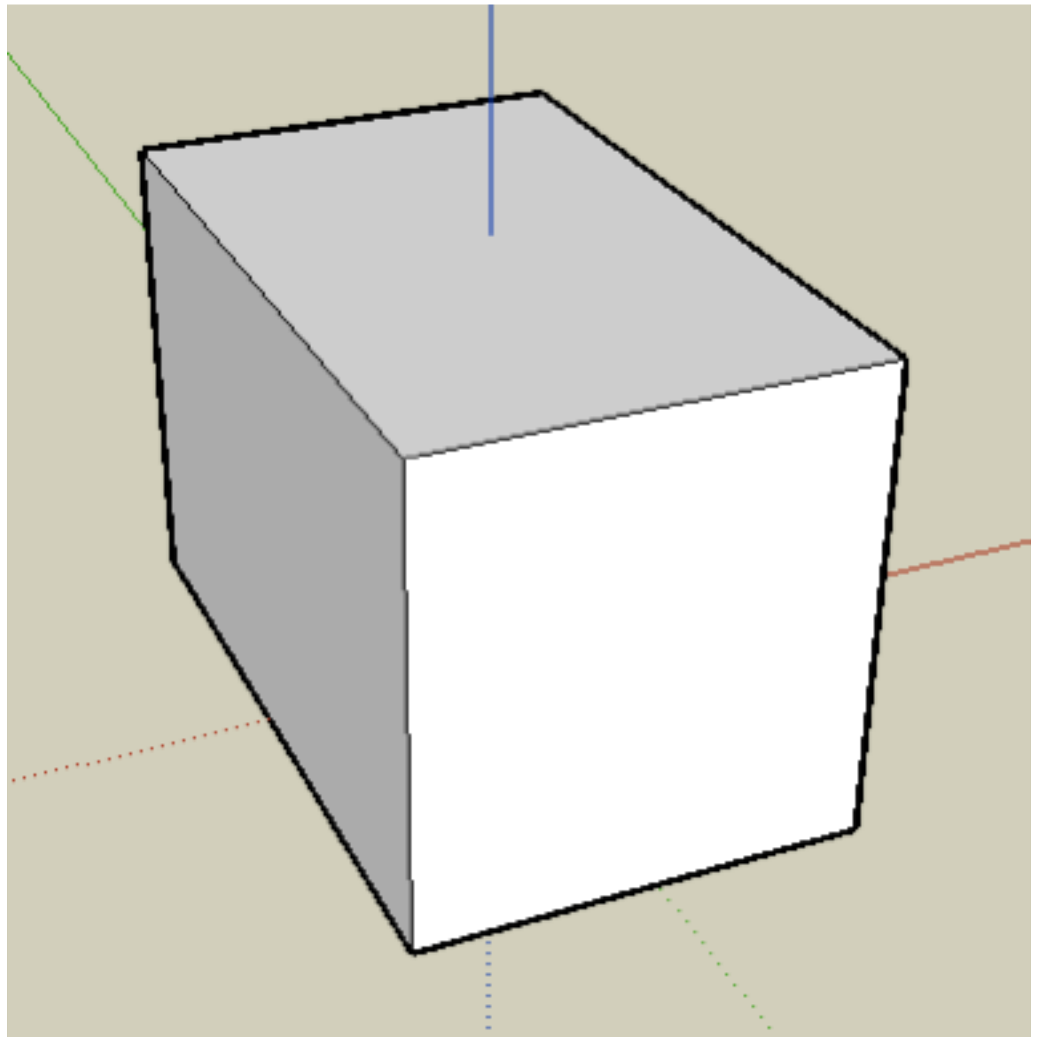
- Simplest way to describe geometric objects
- Can be used directly by some renderers (e.g., Ray tracing)
- Can be transformed into polygons easily (Tesselation)
- Can be transformed into Voxels easily
- Useful for creating simple block world models

- Supported in many frameworks of different levels
  - VRML/X3D, Java 3D
  - OpenGL, WebGL, JOGL



Box

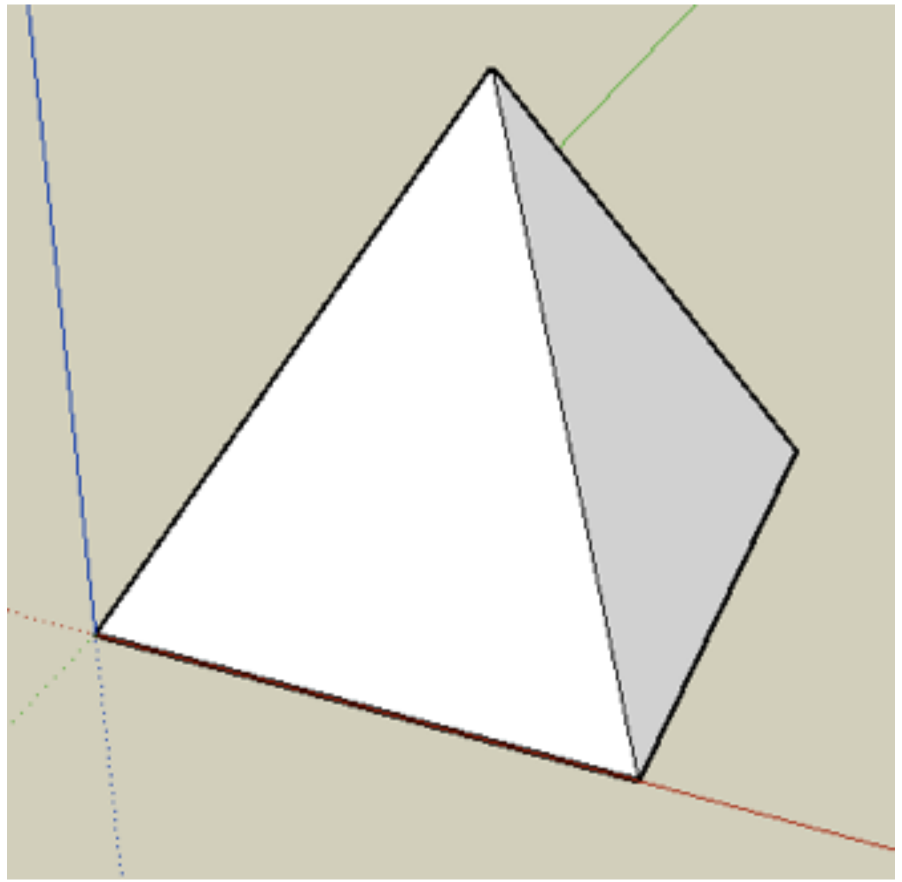
- Described by (width, length, height)
- Origin usually in the center
- 8 points, 12 edges, 6 rectangles, 12 triangles

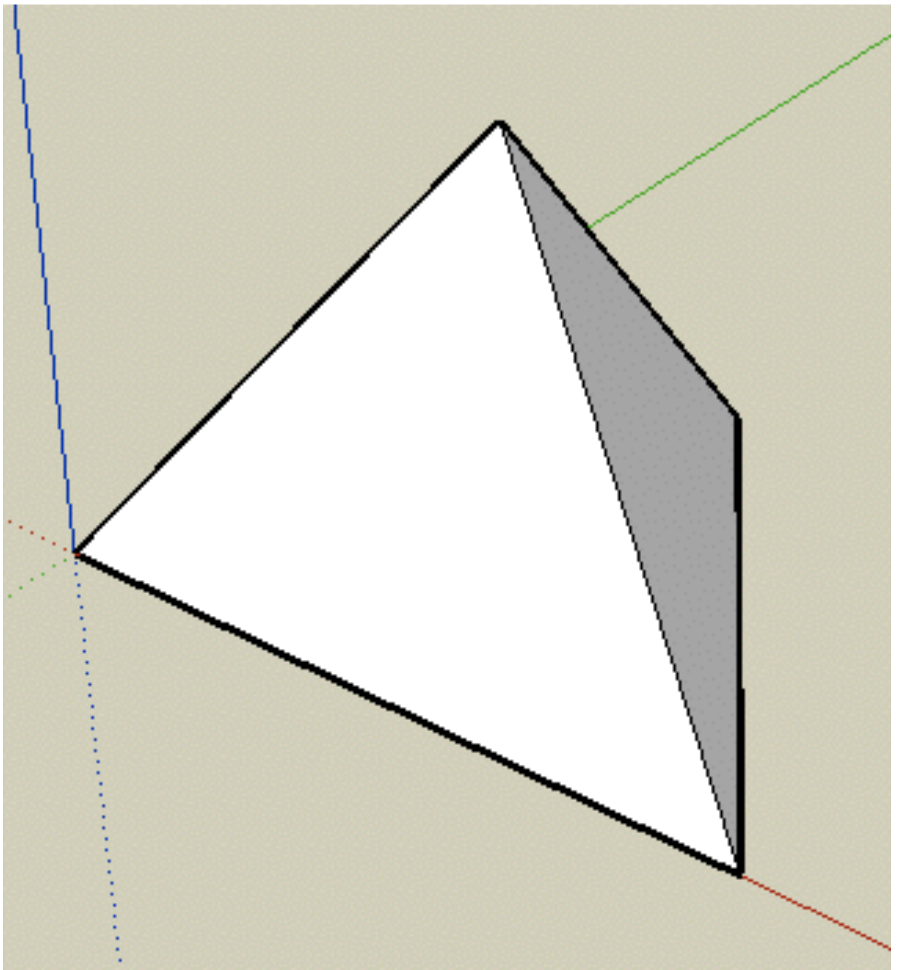


Pyramid, Tetrahedron *(Tetraeder)*

- Basis of pyramid = rectangle
- given by (width, length, height)
- 5 points, 8 edges, 6 triangles

- Basis of tetrahedron = triangle
- given by (width, length, height)
- 4 points, 6 edges, 4 triangles,



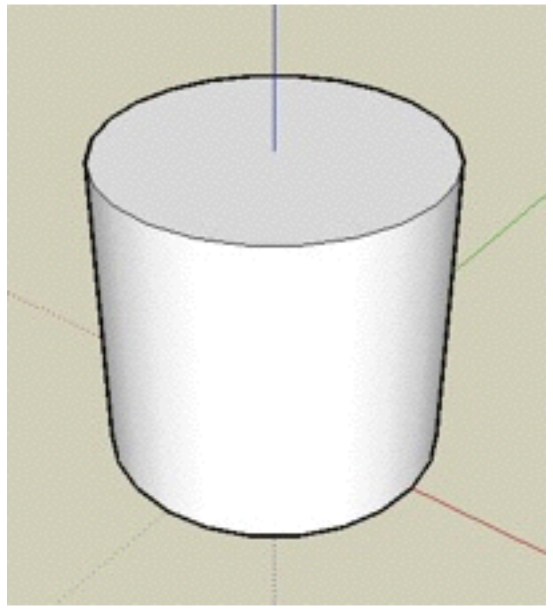


Generalization: Polyhedra

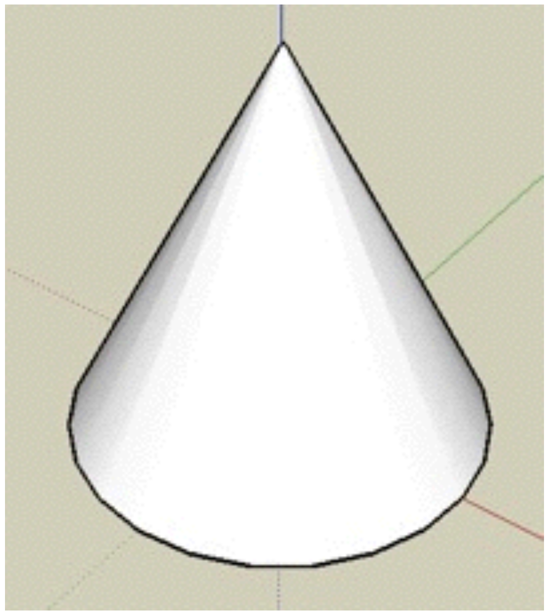
- Polyhedron *(Polyeder)*:
  - Graphical object where a set of surface *polygons* separates the interior from the exterior
  - Most frequently used and best supported by hardware: surface triangles
  - Representation: Table of
    - Vertex coordinates
    - Additional information, like surface normal vector for polygons
- Regular polyhedra: Five Platonic regular polyhedra exist
  - Tetrahedron *(Tetraeder)*
  - Hexahedron, Cube *(Hexaeder, Würfel)*
  - Oktahedron *(Oktaeder)*
  - Dodekahedron *(Dodekaeder)*
  - Icosahedron *(Ikosaeder)*

http://www.aleakybos.ch/

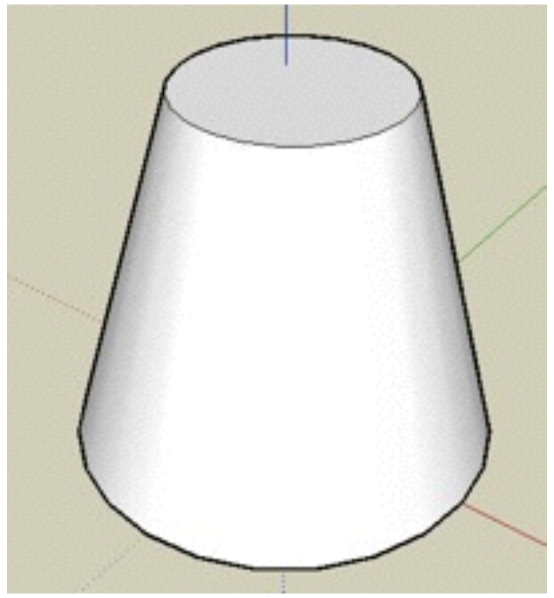
Cylinder, cone, truncated cone



- Cylinder given by (radius, height)
- Number of polygons dep. on tesselation



- Cone given by (radius, height)
- Number of polygons dep. on tesselation

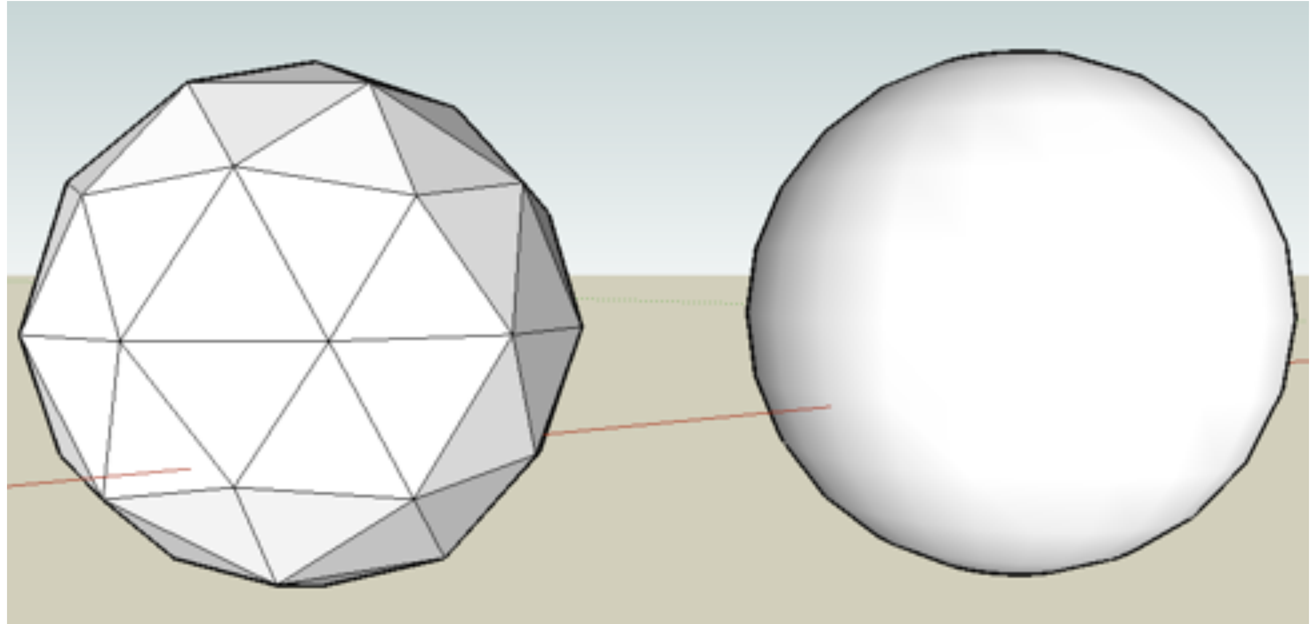


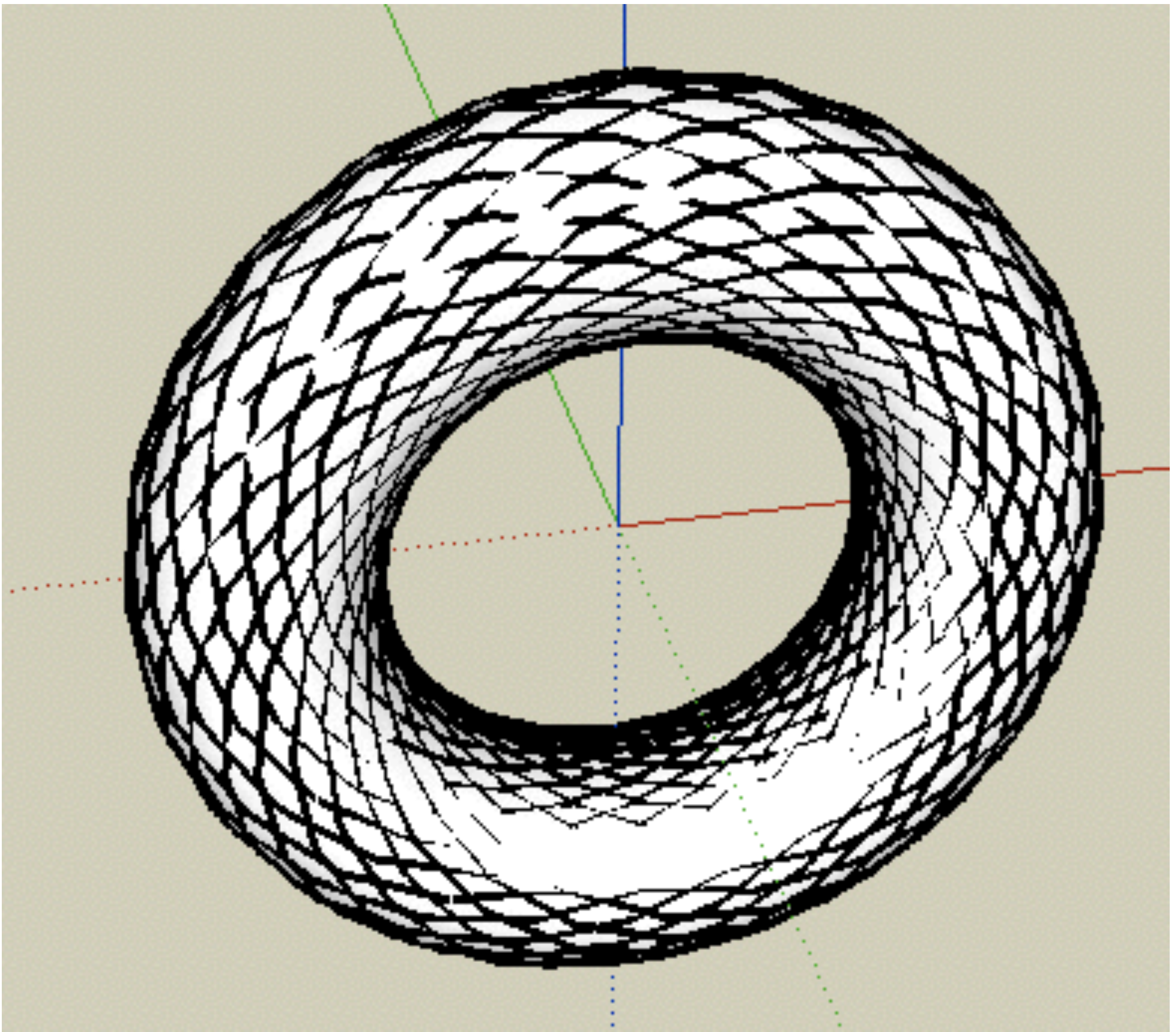
- Truncated cone given by (r1, r2, height)
- Number of polygons dep. on tesselation

- Q: Which of these would you rather have if you only had one available?

Sphere, Torus

- Sphere is described by (radius)
- Torus is defined by (radius1, radius2)
- Number of polygons dep. on tesselation





Geometric Primitives: Summary

- Not all of these exist in all graphics packages
- Some packages define additional primitives (dodecahedron, teapot...;-)

- Practically the only way to model in a text editor
- Can give quite accurate models
- Extremely lean! Very few polygons

- Think of application areas even in times of powerful PC graphics cards!
  –
  –
  –

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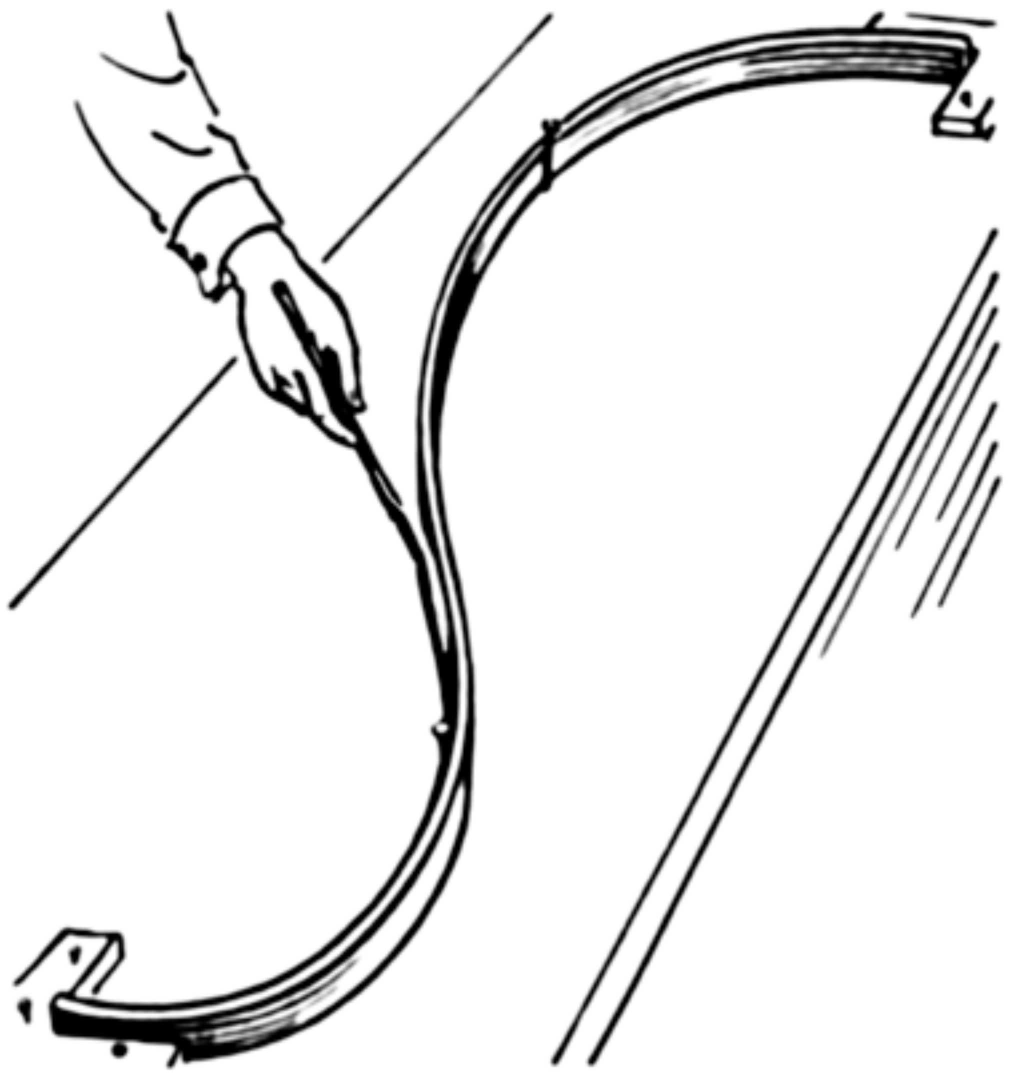
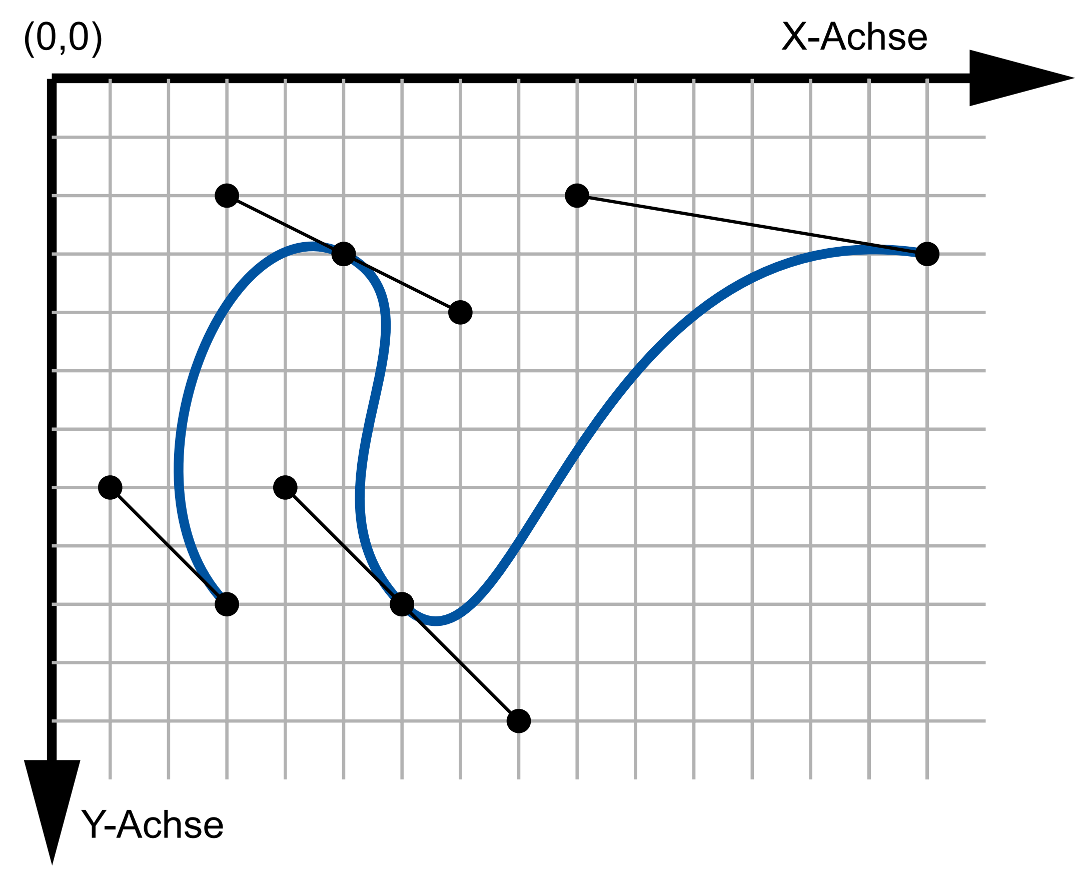
Interpolation Curves, Splines

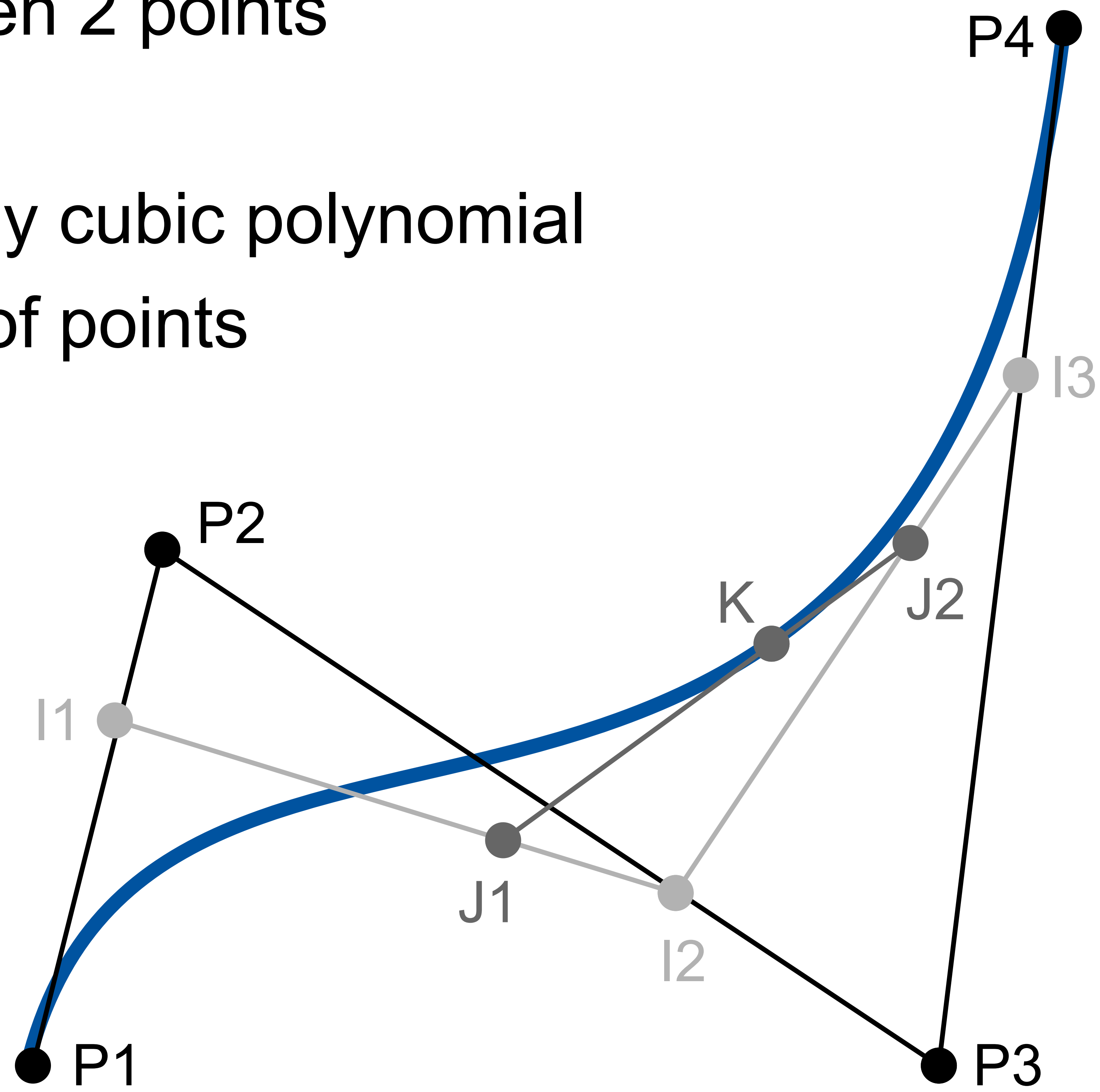
- Original idea: „Spline“ used in ship construction to build smooth shapes:
  - Elastic wooden band
  - Fixed in certain positions and directions
  - Mathematically simulated by interpolation curves
  - Piecewise described by polynomials
- Different types exist
  - Natural splines
  - Bézier curves
  - B-Splines
- Control points may be on the line or outside of it.
  - All on the line for a natural spline



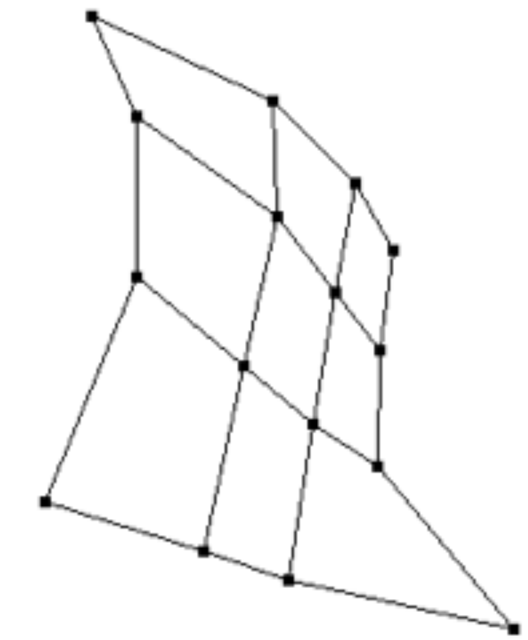


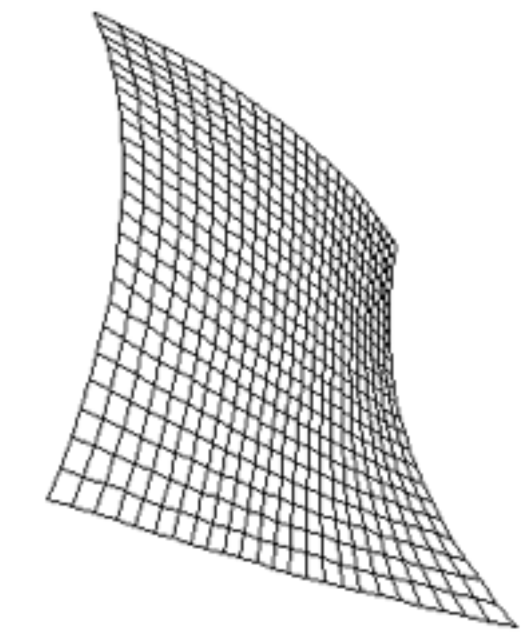
Bézier Curves (and de-Casteljau Algorithm)

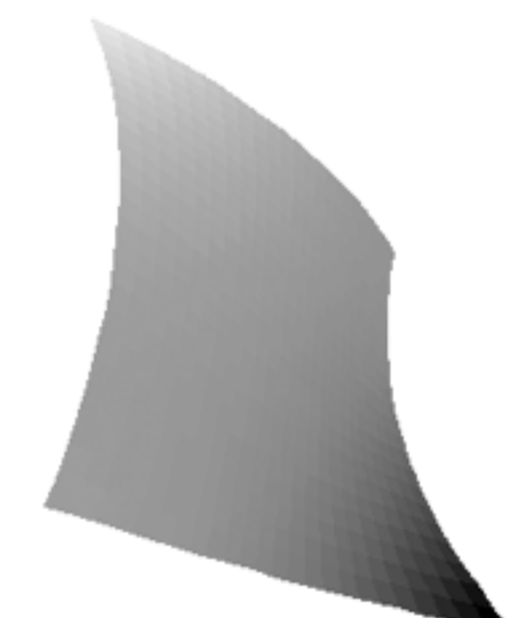
- Bézier curves first used in automobile construction (1960s, Pierre Bézier – Renault, Paul de Casteljau – Citroën)
- Degree 1: straight line interpolated between 2 points
- Degree 2: quadratic polynomial
- Degree 3: cubic Bézier curve, described by cubic polynomial
- Curve is always contained in convex hull of points
- Algorithm (defines line recursively):
  - Choose *t* between 0 and 1
  - I1: Divide line between P1 and P2 as *t* : (1–*t*)
  - I2, I3: Repeat for all Ps *(one segment less!)*
  - J1, J2: Repeat for I1, I2, I3 (same *t*)
  - K: Repeat for J1, J2 *(single point!)*
  - Bézier curve: all points K for *t* between 0 and 1
- see http://goo.gl/m7Z1Y (Dominik Menke)



Bézier Patches



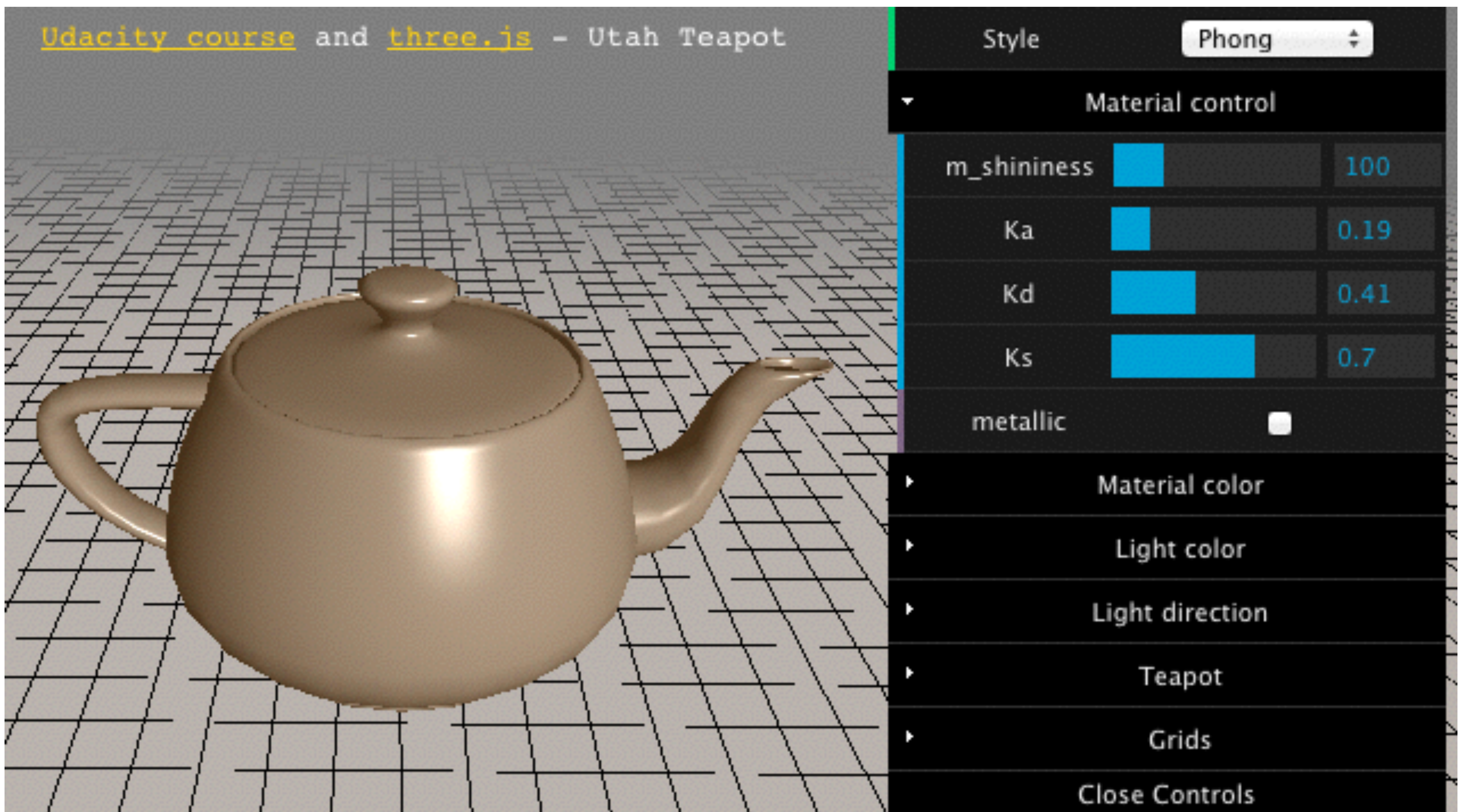




- Combine 4 Bézier curves along 2 axes
- Share 16 control points
- Results in a smooth surface
- Entire surface is always contained within the convex hull of all control points
- Border line is fully determined by border control points
- Several patches can be combined
  - connect perfectly if border control points are the same.
- Advantage: move just one control point to deform a larger surface...
- Other interpolation surfaces based on other curves
  - Generalization of Bézier idea: B-splines
  - Further generalization: Non-uniform B-splines
  - Non-uniform rational B-splines (NURBS) *(supported by OpenGL GLU)*

Interpolation in OpenGL (Bezier Example)

- Utah teapot
  - Martin Newell, 1975
  - 306 vertices
  - 32 bicubic Bézier surface patches



http://www.realtimerendering.com/teapot/

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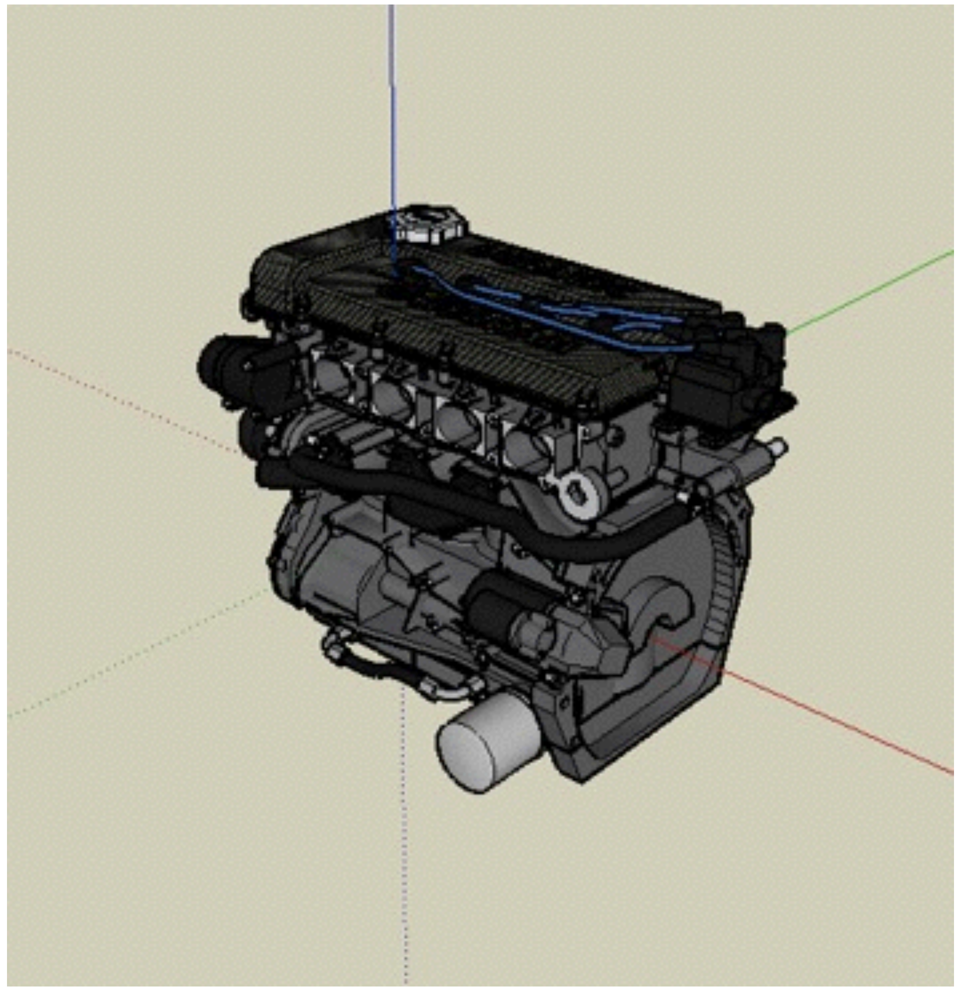
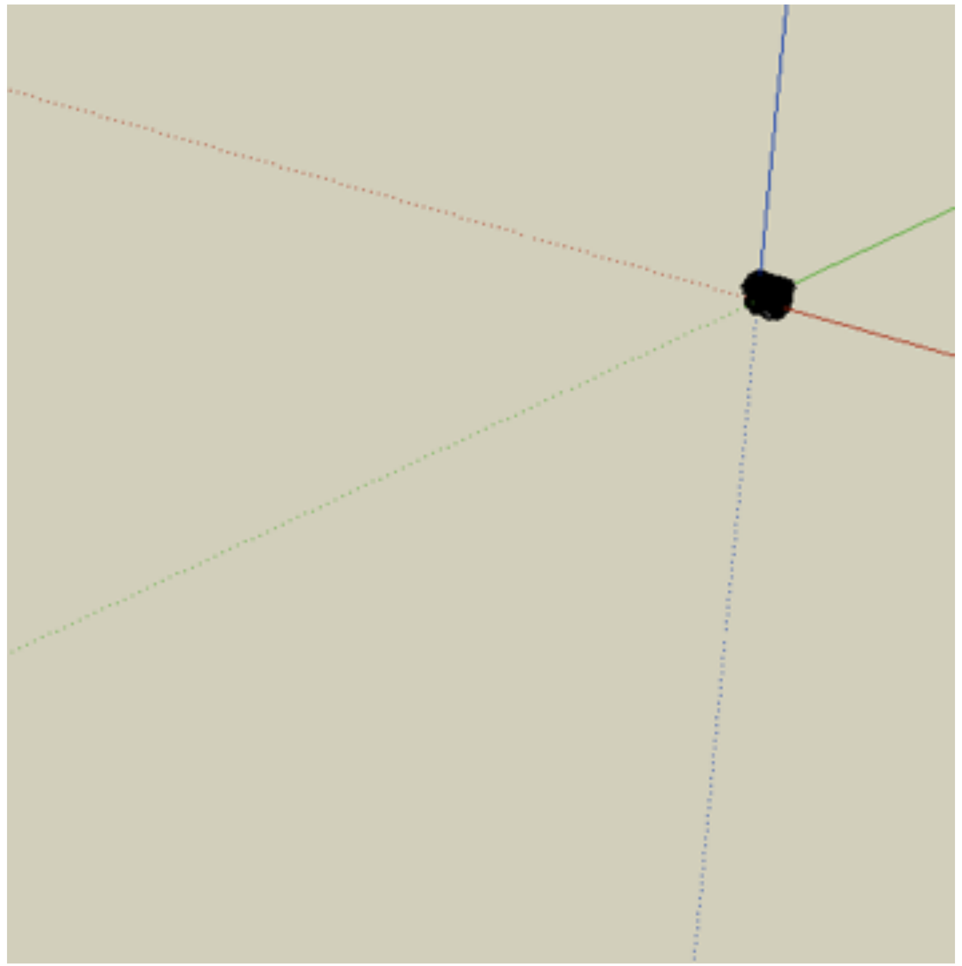
Levels of Detail

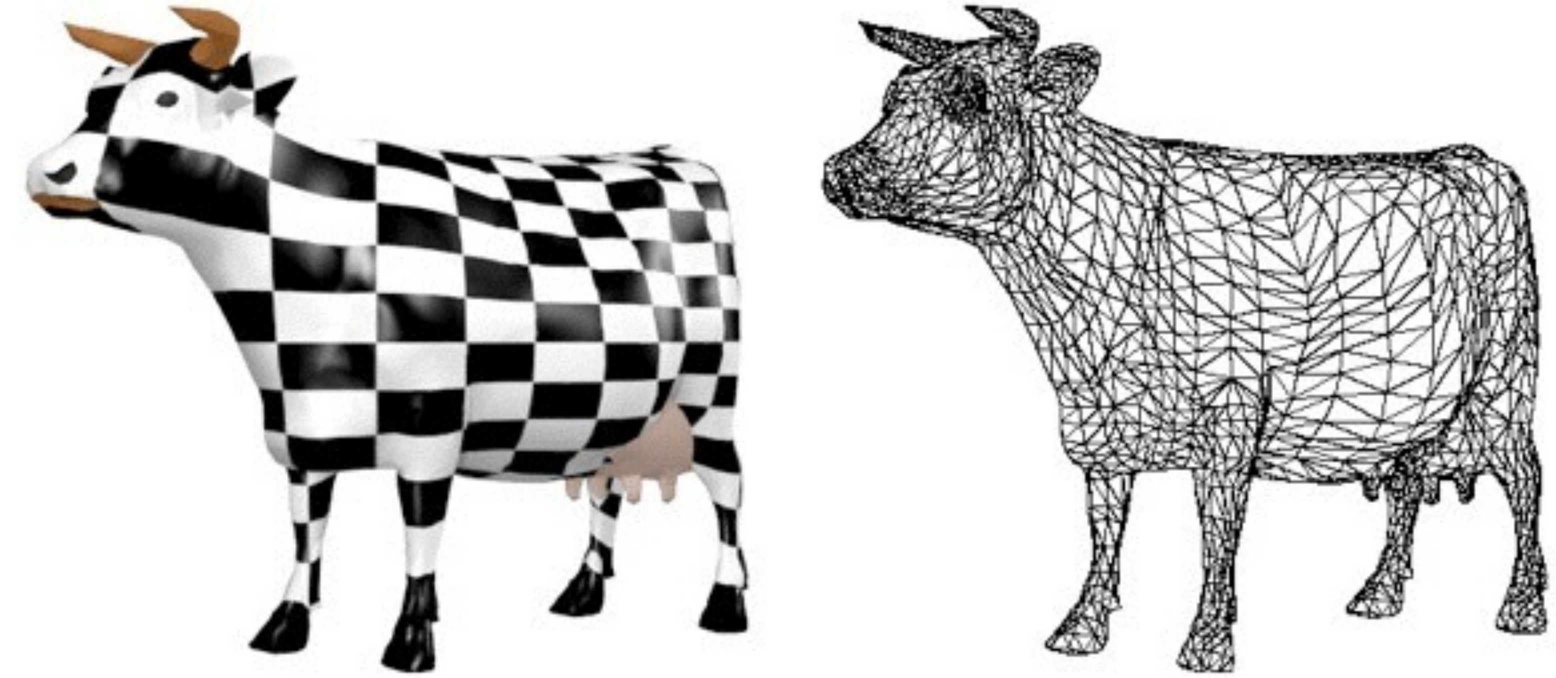
- Assume you have a very detailed model
- from close distance, you need all polygons
- from a far distance, it only fills a few pixels
- How can we avoid drawing all polygons?
  -
  -
  -
  -





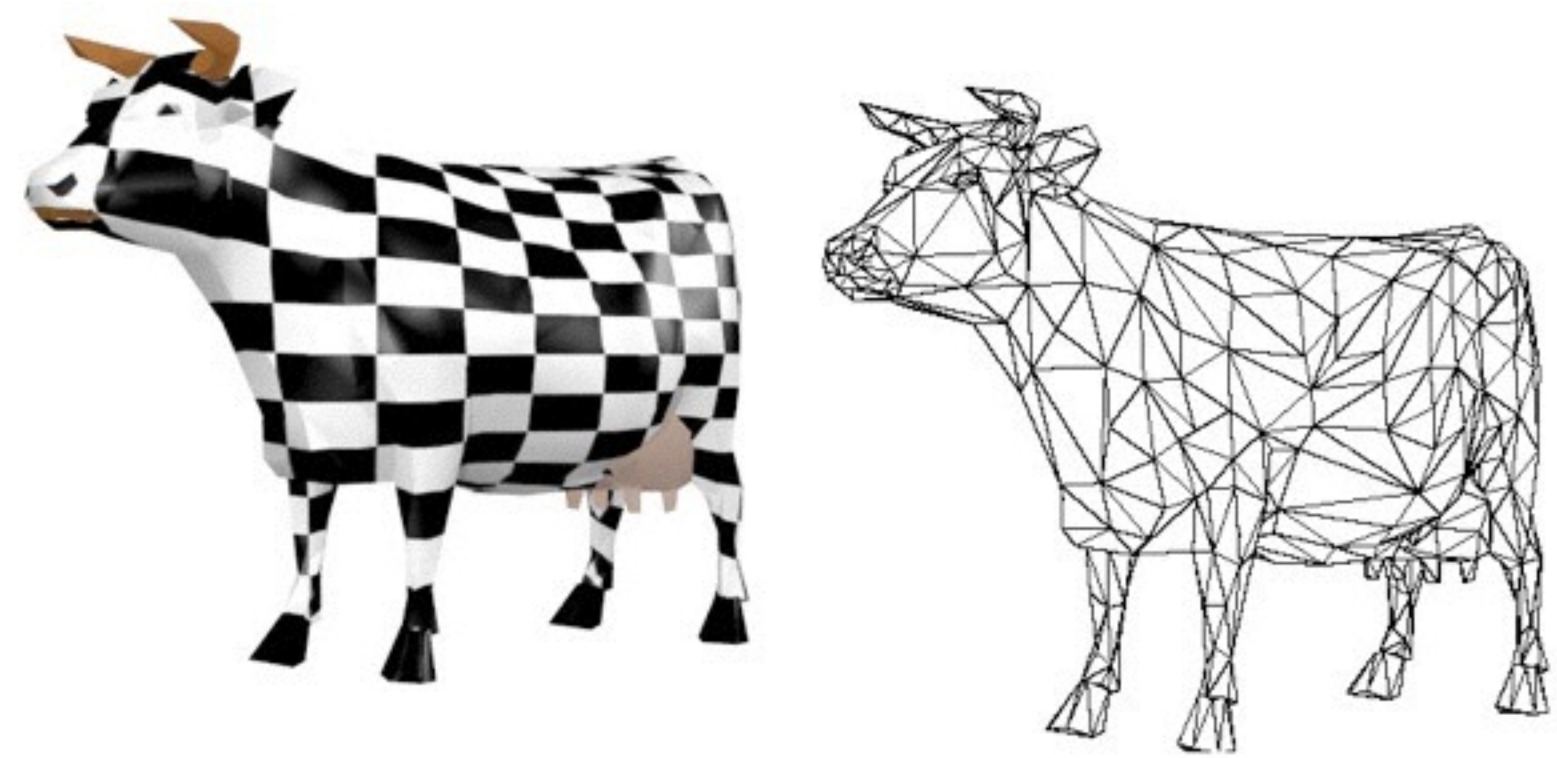
Mesh reduction

- Original: ~5.000 polygons
- Reduced model: ~1.000 polygons
- ==> about 80% reduction



- Very strong reductions possible, depending on initial mesh

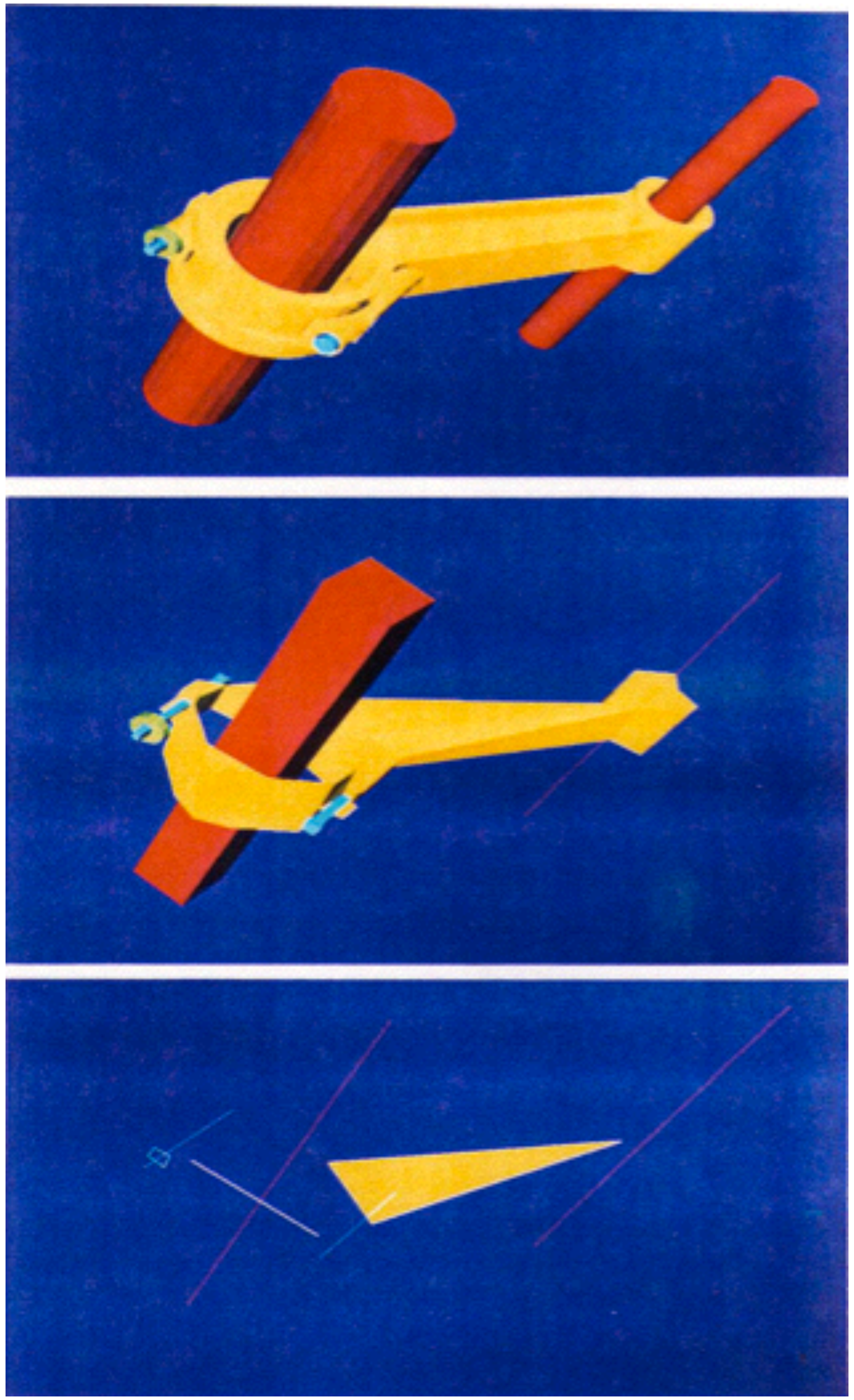
- Loss of shape if overdone



http://www.okino.com/conv/polygon_reduction/geoman2/polygon_reduction_tutorial1.htm

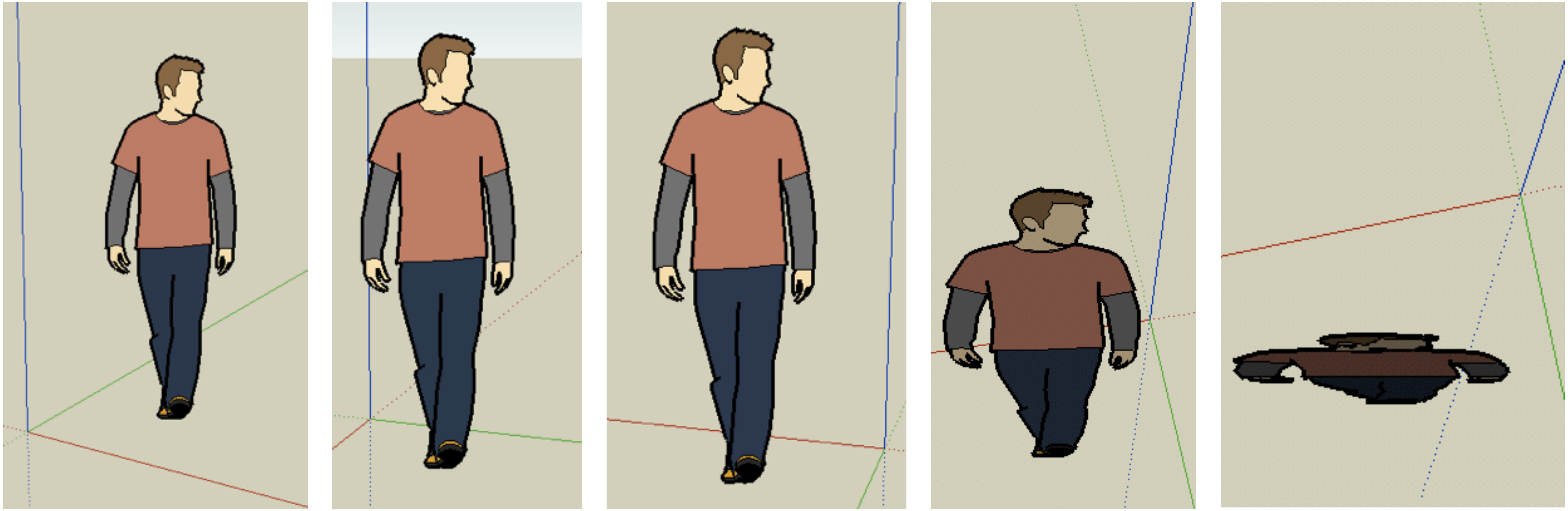
A method for polygon reduction

- Rossignac and Borell, 1992, „Vertex clustering“
- Subdivide space into a regular 3D grid
- For each grid cell, melt all vertices into one
  - Choose center of gravity of all vertices as new one
  - Triangles within one cell disappear
  - Triangles across 2 cells become edges (i.e. disappear)
  - Triangles across 3 cells remain
- Good guess for the minimum size of a triangle
  - edge length roughly = cell size
- Yields constant vertex density in space
- Does not pay attention to curvature



Billboard

- A flat object which is always facing you
- Very cheap in terms of polygons (2 triangles)
- Needs a meaningful texture
- Example (from SketchUp): guy in the initial empty world rotates about his vertical axis to always face you



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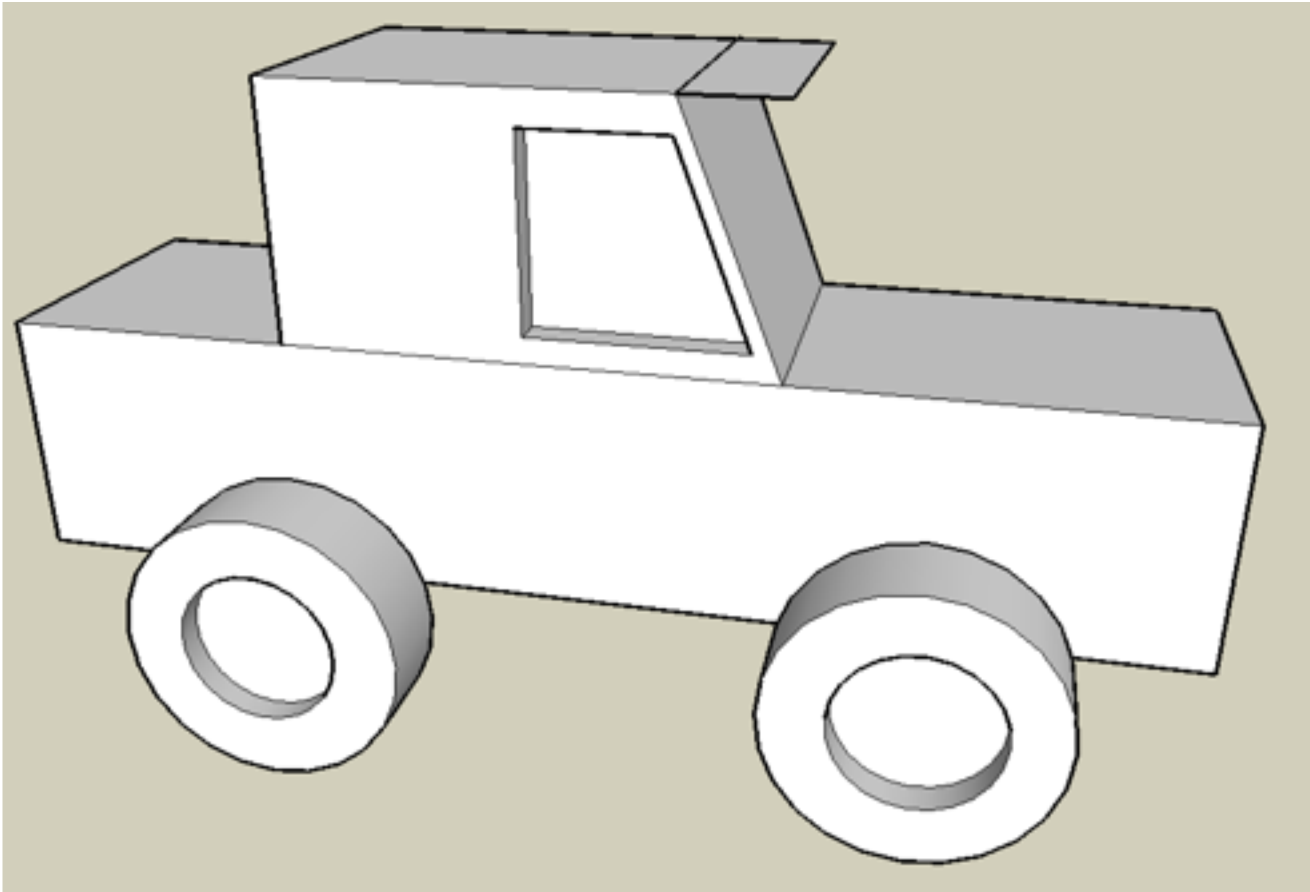
Constructive Solid Geometry

- Basic idea: allow geometric primitives and all sorts of boolean operations for combining them
- Can build surprisingly complex objects
- Good for objects with holes (often the simplest way)
- Basic operations:
  - **Or**: combine the volume of 2 objects
  - **And**: intersect the volume of 2 objects
  - **Not**: all but the volume of an object
  - **Xor**: all space where 1 object is, but not both
- Think about:
  - wheels of this car
  - tea mug
  - coke bottle (Problems??)

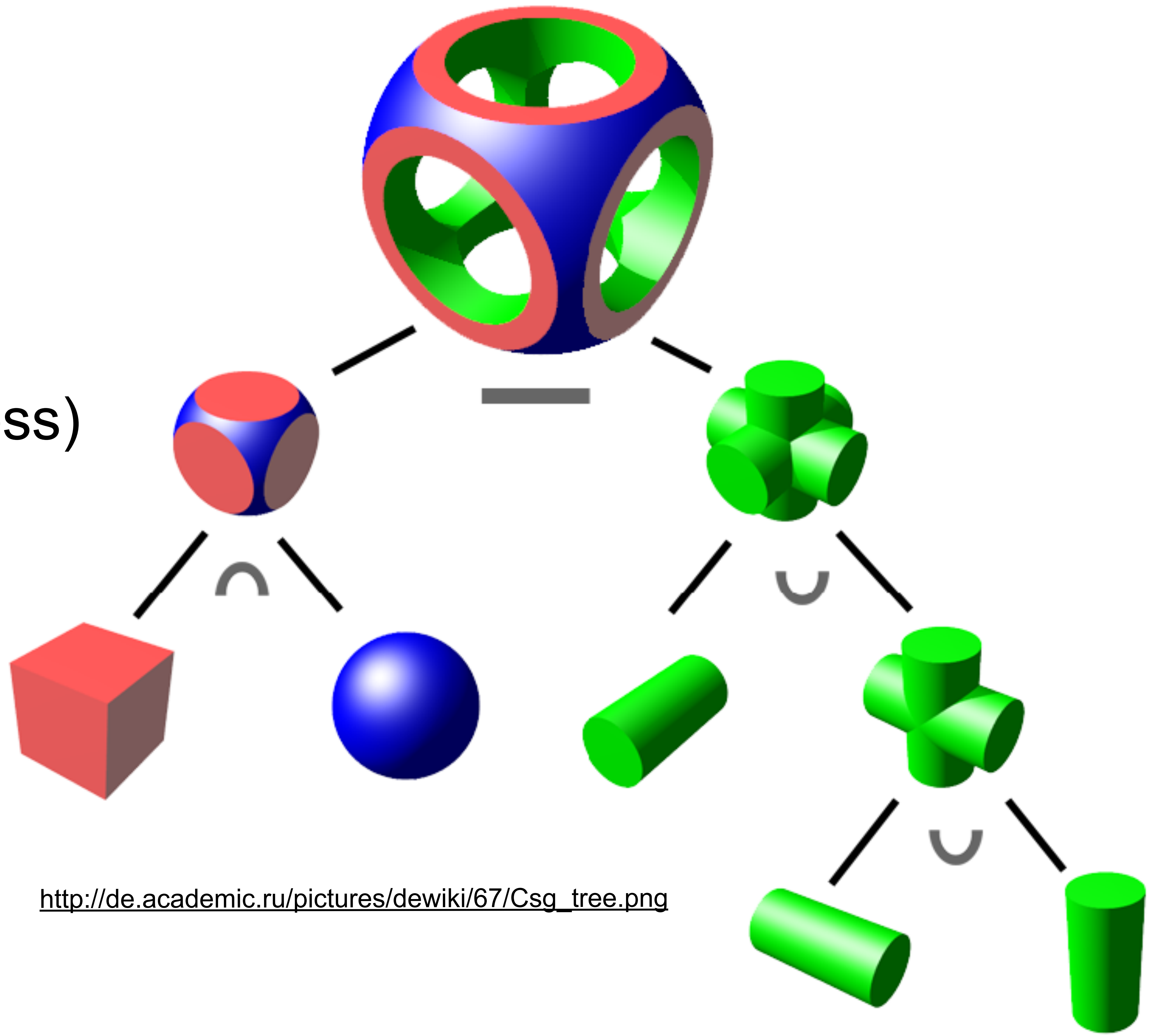


CSG: a complex Example

- rounded_cube =
  cube **And** sphere
- cross =
  cyl1 **Or** cyl2 **Or** cyl3
- result =
  rounded_cube **And** (**Not** cross)

- Think: Are CSG operations associative?
  -

- ...commutative?
  -



http://de.academic.ru/pictures/dewiki/67/Csg_tree.png

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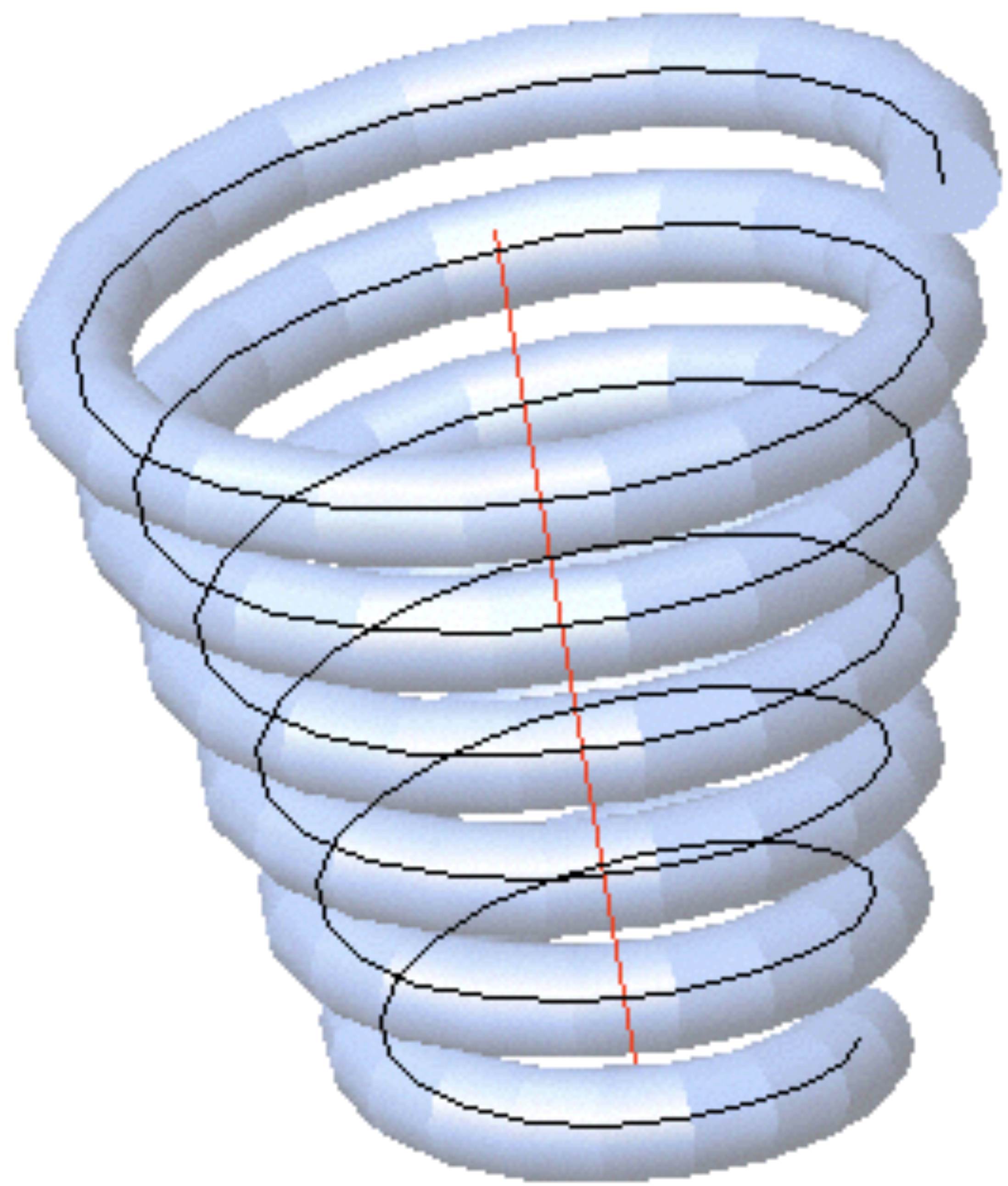
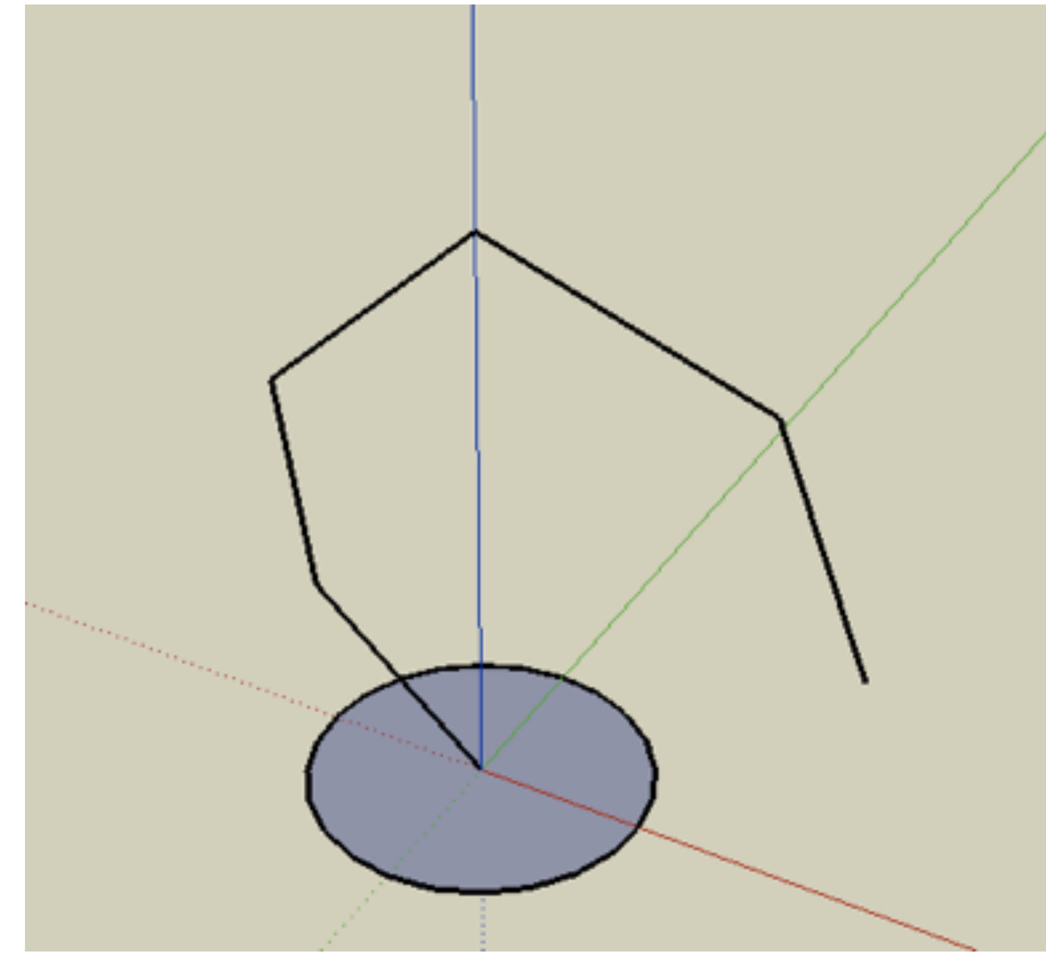
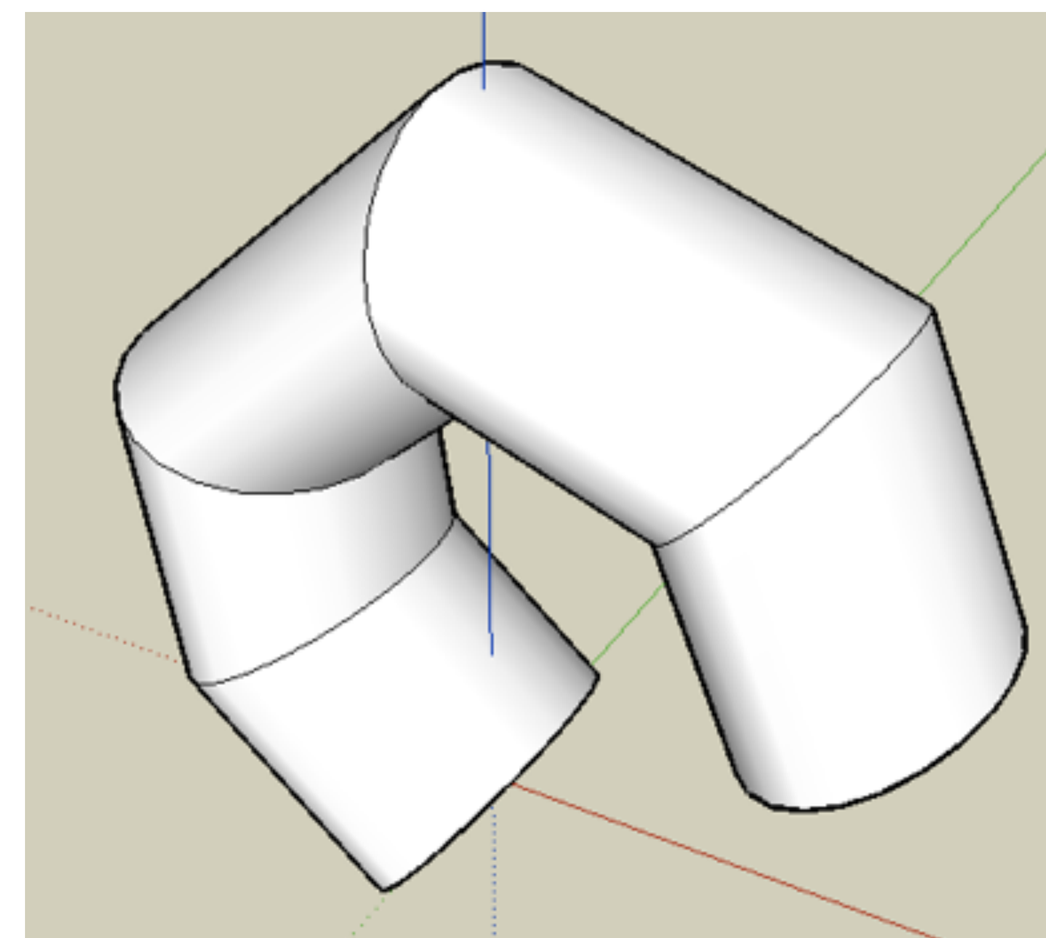
Extrusion (sweep object)

- Move a 2D shape along an arbitrary path
- Possibly also scale in each step



http://www.cadimage.net/cadtutor/lisp/helix-02.gif

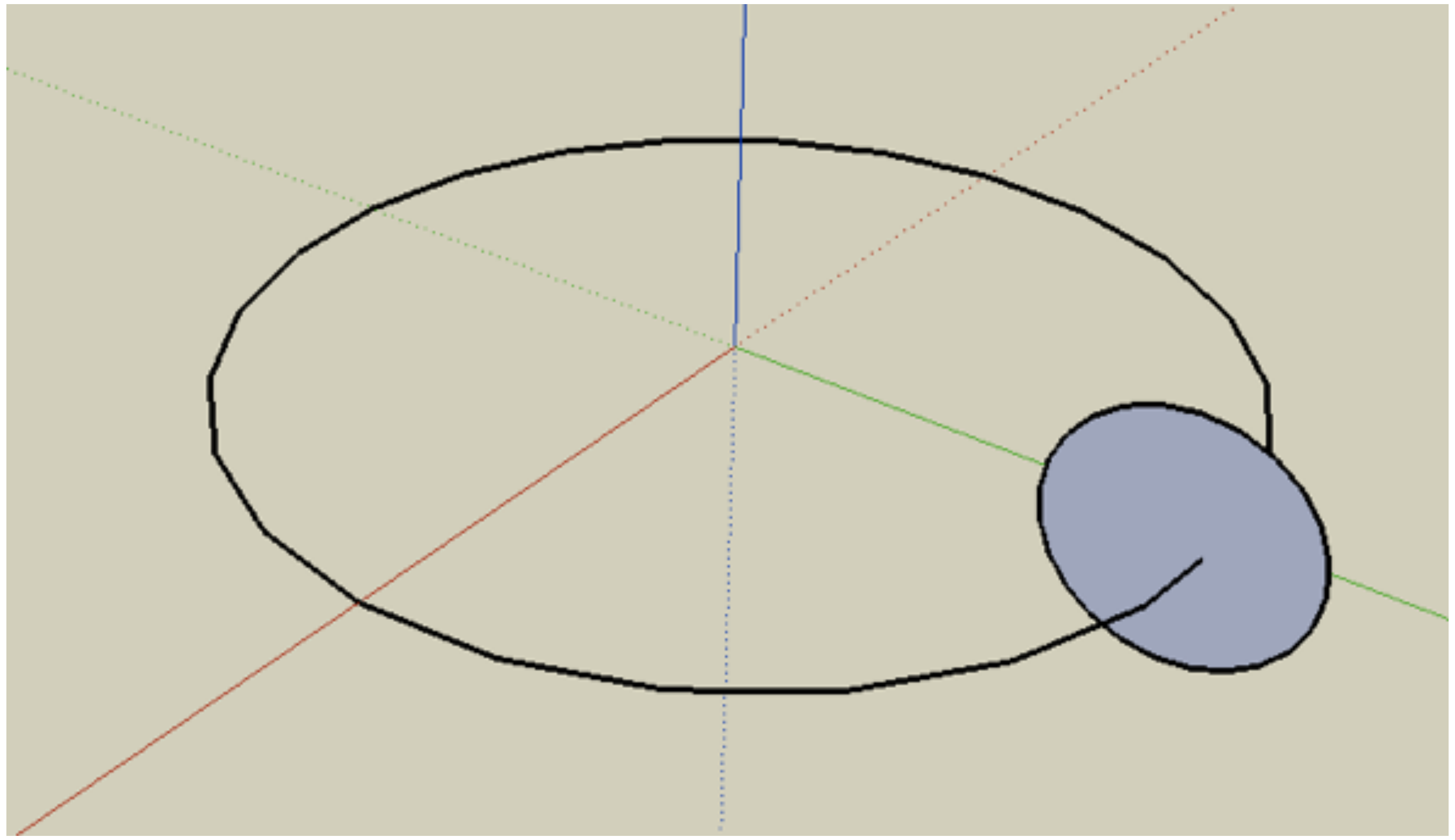


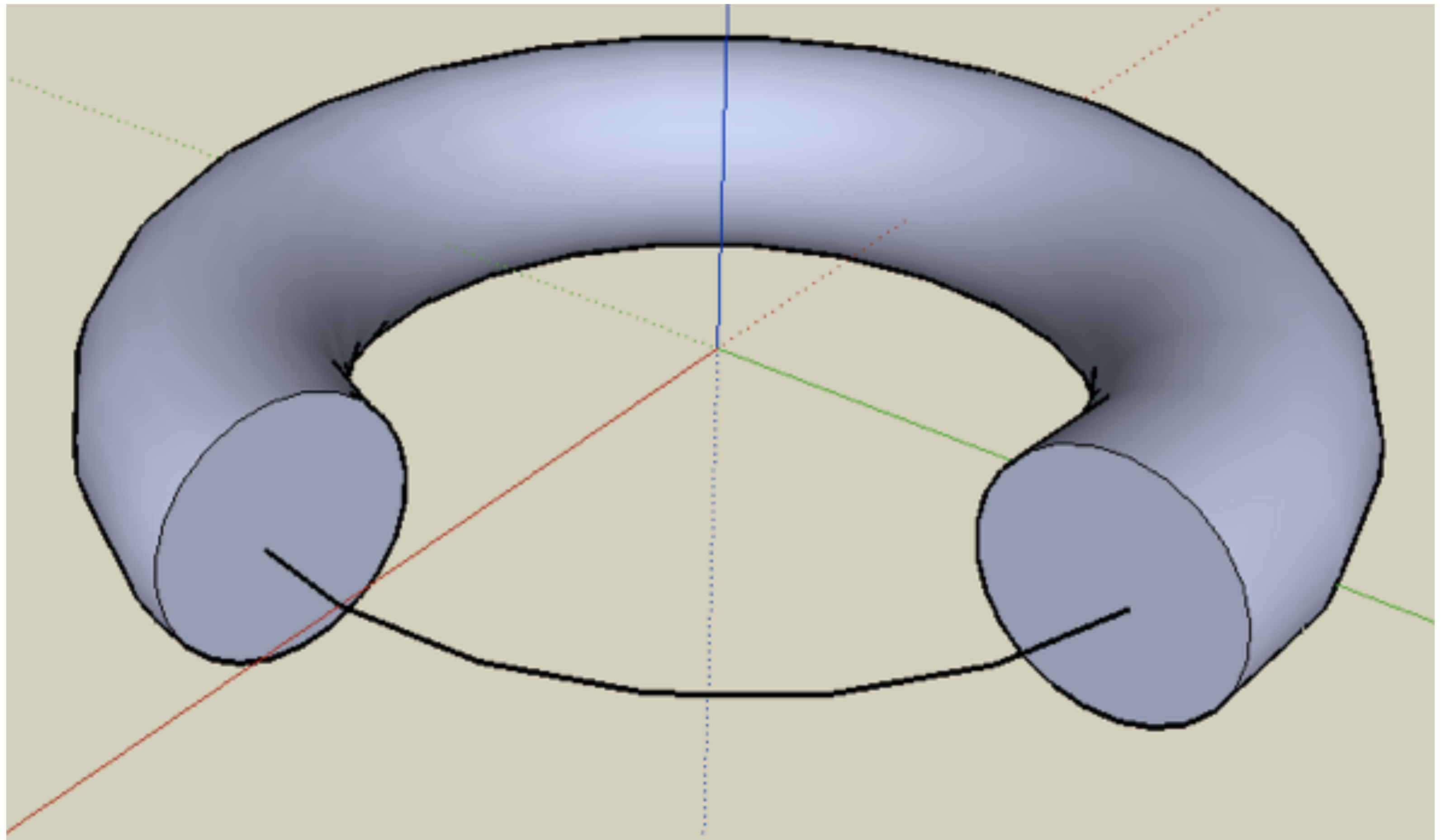


Rotation

- Rotate a 2D shape around an arbitrary axis
- Can be expressed by extrusion along a circle

- How can we model a vase?
  –
  –
  –
- How a Coke bottle?
  –
  –
  –



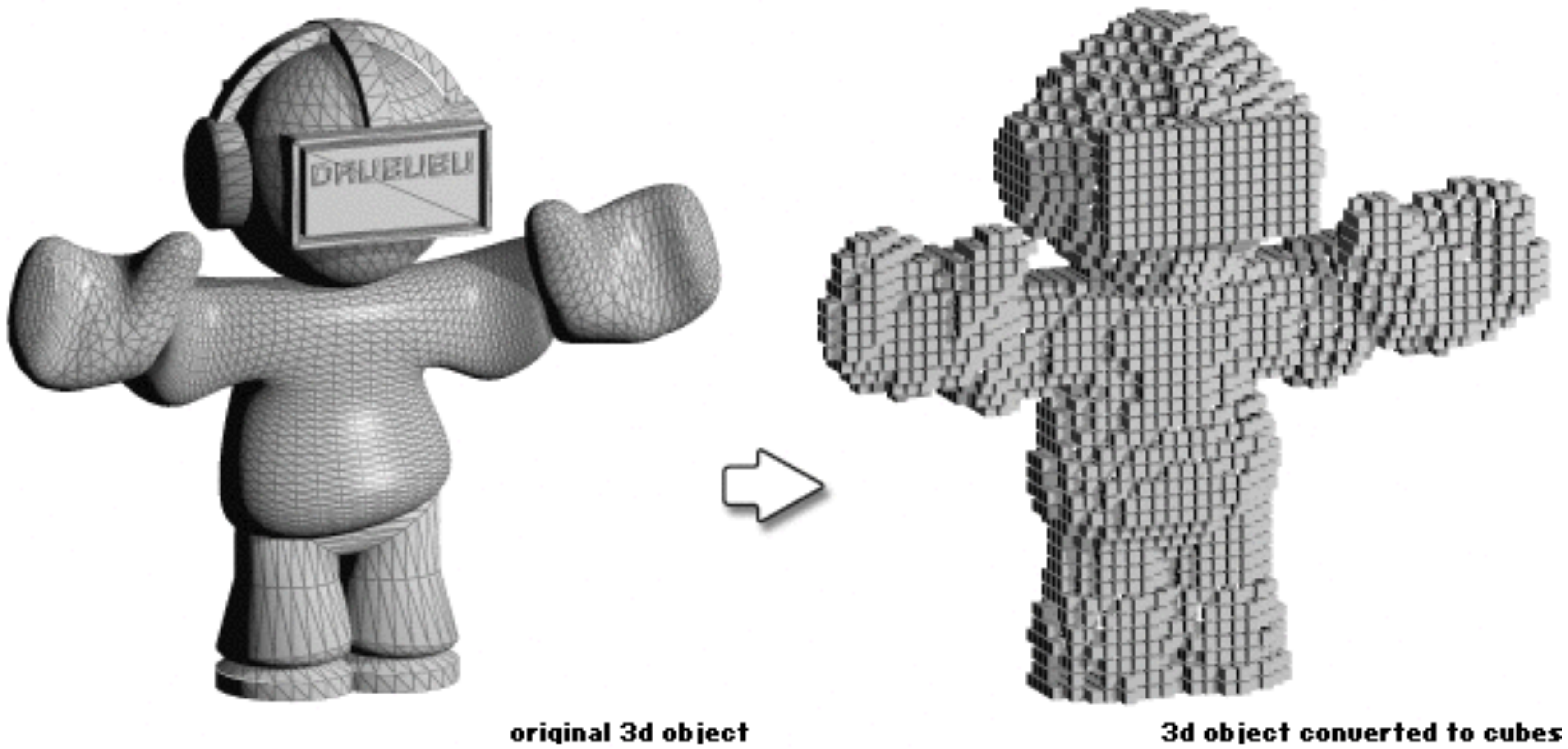


Chapter 3 - 3D Modeling

- Polygon Meshes
- Geometric Primitives
- Interpolation Curves
- Levels Of Detail (LOD)
- Constructive Solid Geometry (CSG)
- Extrusion & Rotation
- Volume- and Point-based Graphics

Voxel data

- „Voxel“ = „Volume“ + „Pixel“, i.e., voxel = smallest unit of volume
- Regular 3D grid in space
- Each cell is either filled or not
- Memory increases (cubic) with precision



- Easily derived from CSG models
- Also the result of medical scanning devices
  - MRI, CT, 3D ultrasonic

- Volume rendering = own field of research
- Surface reconstruction from voxels

http://www.drububu.com/tutorial/voxels.html

Point-based graphics

- Objects represented by point samples of their surface („Surfels“)
- Each point has a position and a color
- Surface can be visually reconstructed from these points
  - purely image-based rendering
  - no mesh structure
  - very simple source data (x,y,z,color)

- Point-data is acquired e.g., by 3D cameras
- Own rendering techniques
- Own pipeline
- ==> own lecture ;-)

http://www.crs4.it/vic/data/images/img-exported/stmatthew_4px_full_shaded2.png