Tutorial 7

Illumination

Computer Graphics

Summer Semester 2020

Ludwig-Maximilians-Universität München

Agenda

- Whitted-style Ray Tracing
- Monte Carlo Ray Tracing
  - Monte Carlo Integration
  - Path Tracing
  - Light Sampling
  - Direct & Indirect Illumination
- Epilogue

Tutorial 7: Illumination

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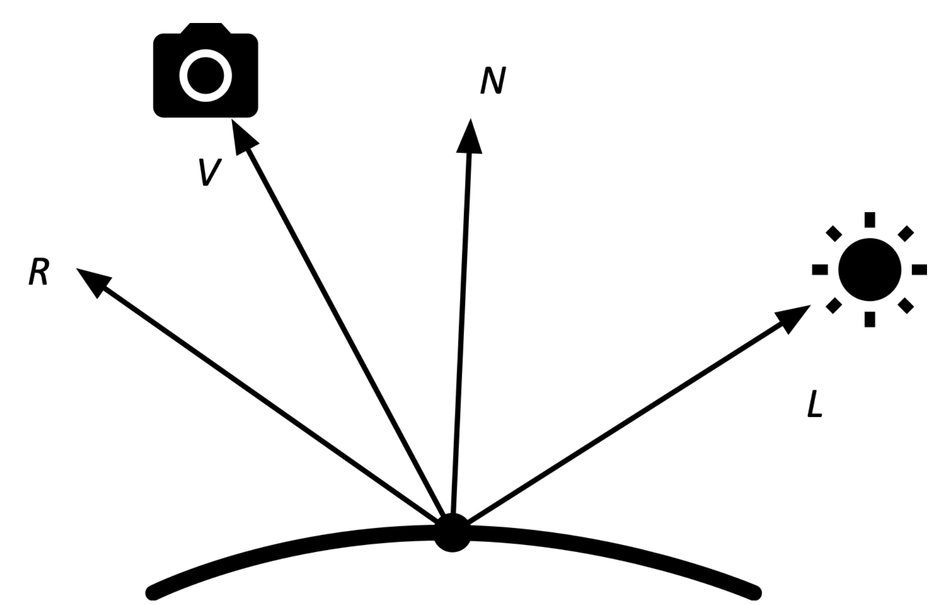
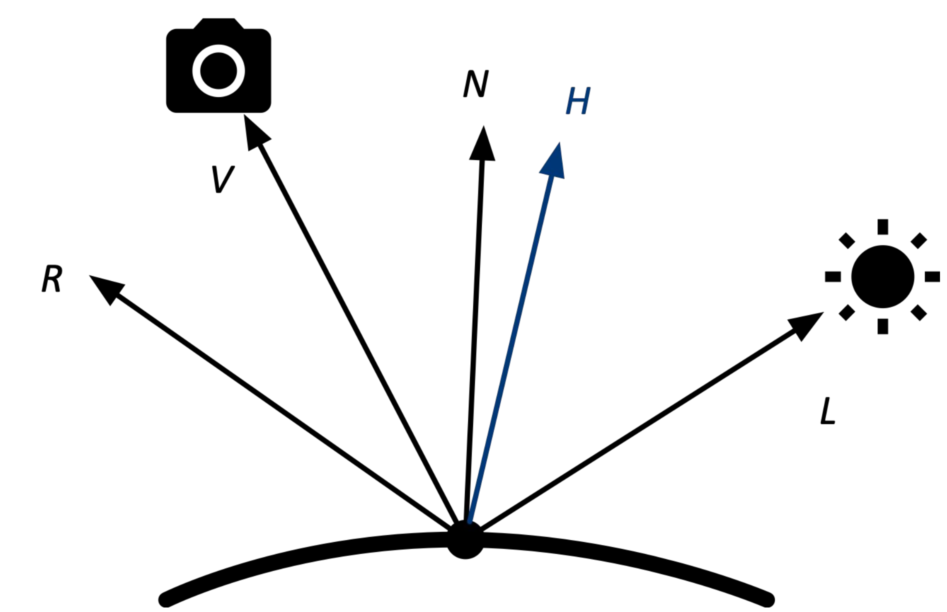
Revisit: Phong and Blinn-Phong's (Local) Model

$$L_{\text{Phong}} = L_a + L_d + L_s = k_a I_a + k_d I_d \max\left(0, \mathbf{N} \cdot \mathbf{L}\right) + k_s I_s \max(0, \mathbf{R} \cdot \mathbf{V})^p$$

$$L_{\text{Blinn-Phong}} = L_a + L_d + L_s' = k_a I_a + k_d I_d \max\left(0, \mathbf{N} \cdot \mathbf{L}\right) + k_s I_s \max(0, \mathbf{N} \cdot \mathbf{H})^p$$





What do we do about reflected light?

What do we do about shadows in these models?

…

Intuitively, they totally do not match the real world!

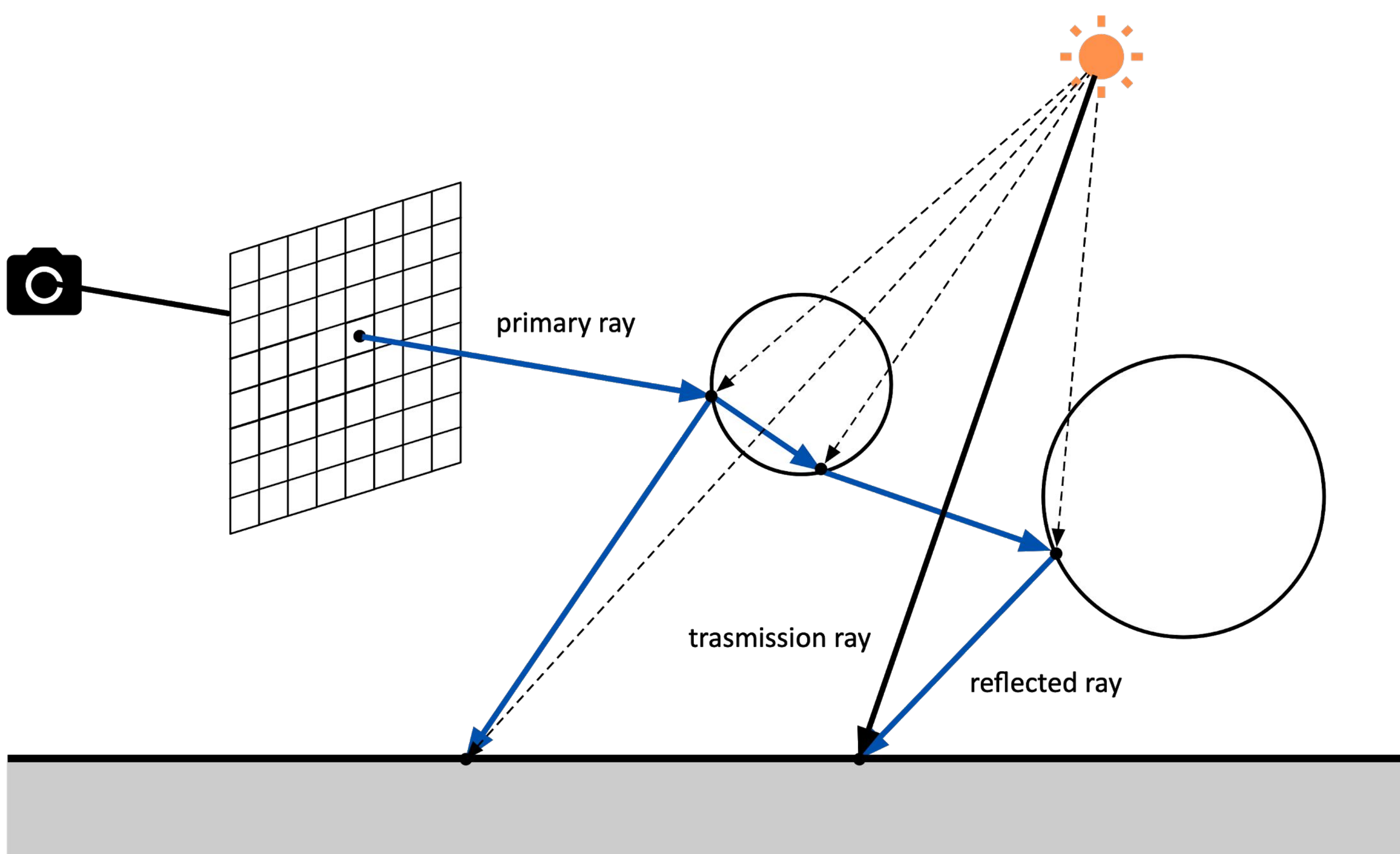
Whitted-style: An Improved (Global) Illumination Model

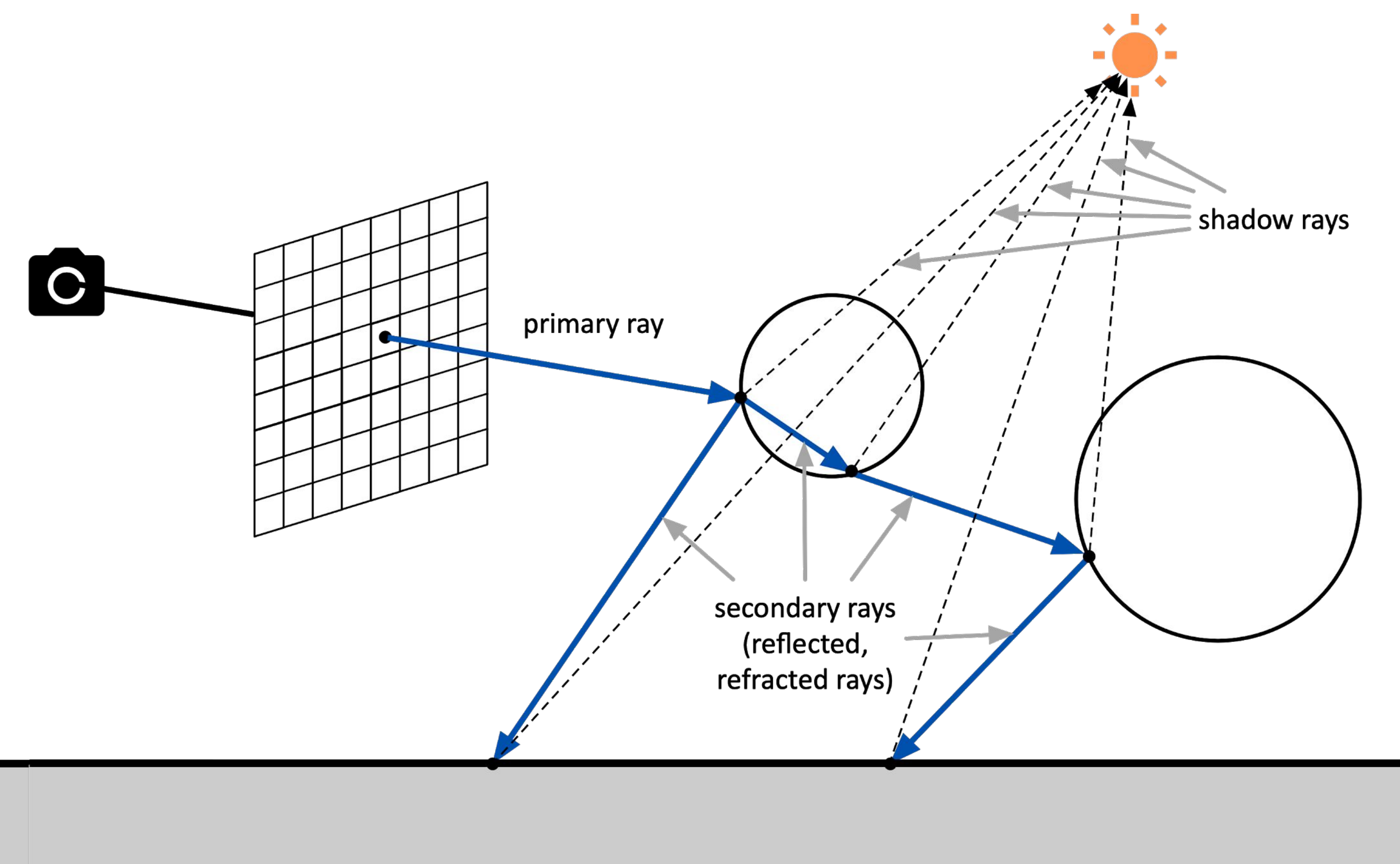
Simple idea: ***Ray tracing***. Trace a ray's path, sum up the intensity

Intensity from a reflected ray **Intensity from a transmission ray**

$$L_{\text{Whitted}} = k_a L_a + k_d I_d \mathbf{N} \cdot \mathbf{L} + k_s S + k_t T$$

Ambient Term **Diffuse Term** **Whitted Term**

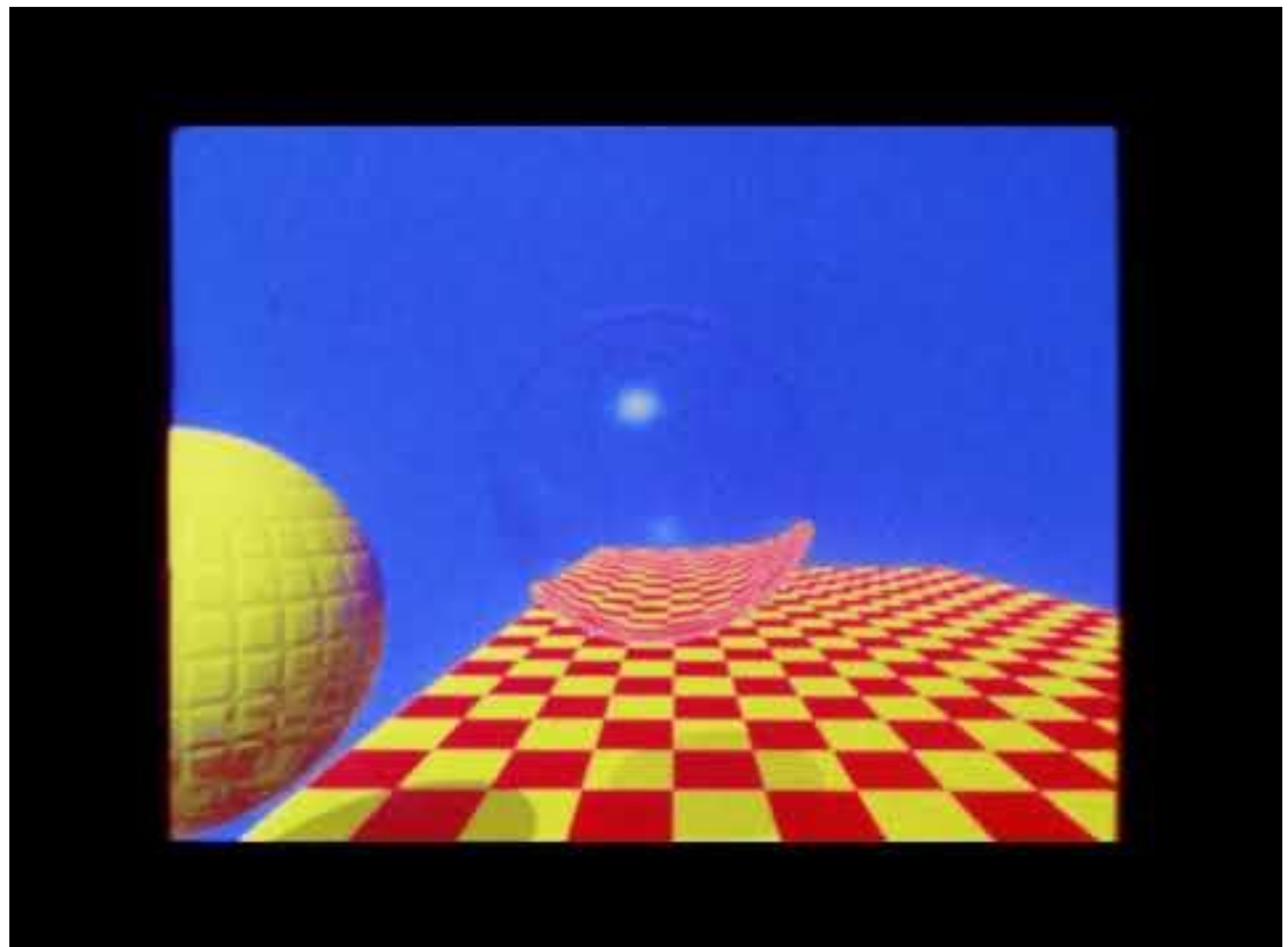




Turner Whitted. 1980. An improved illumination model for shaded display. Commun. ACM 23, 6 (June 1980), 343–349. DOI:https://doi.org/10.1145/358876.358882

Whitted-style Ray Tracing (1980)

- Always perform specular reflections / refractions
- Stop bouncing at diffuse surface



https://blogs.nvidia.com/blog/2018/08/01/ray-tracing-global-illumination-turner-whitted/

Ray Tracing in Practice: Performance Issue

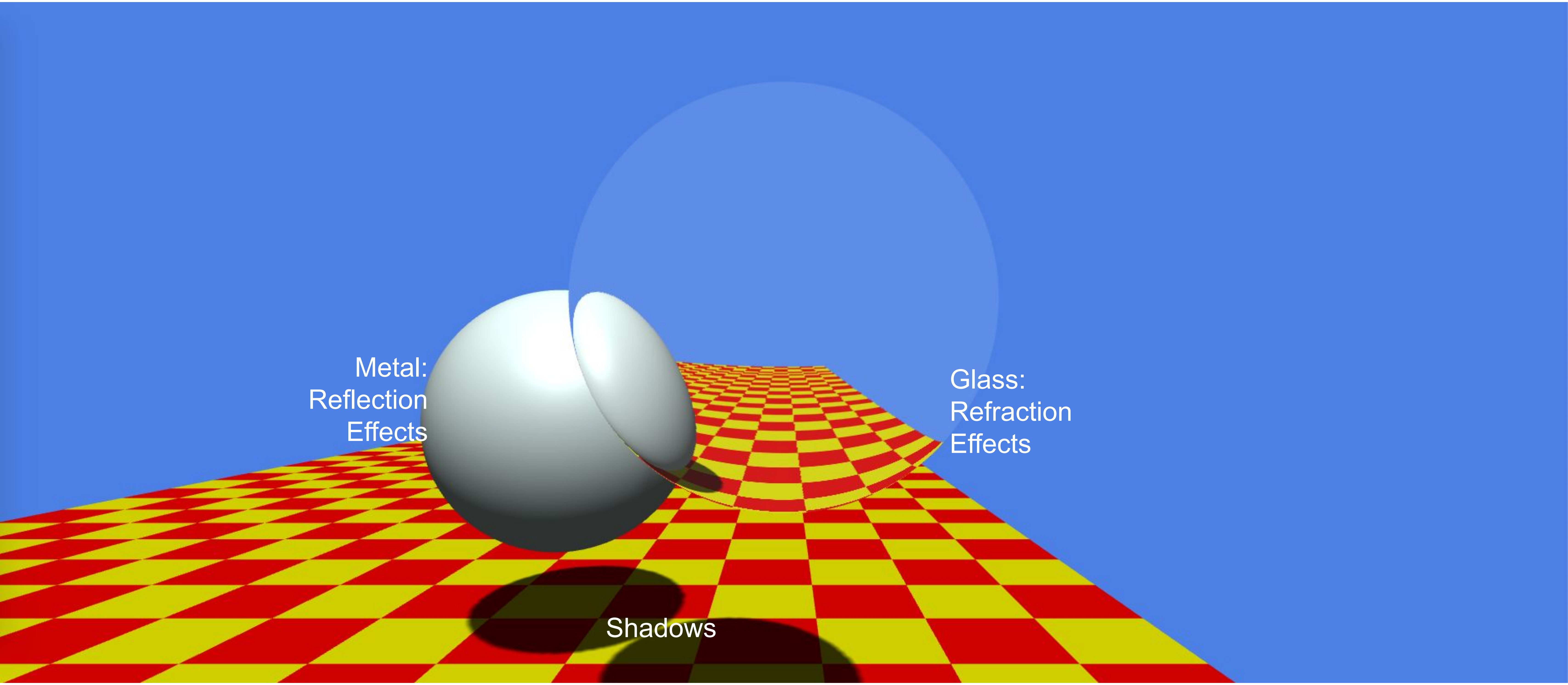
- Naive way to trace a ray
  - Exhaustively test ray intersections with every object (each object has many triangles)
  - Which object will be intersected?
  - Which triangle will be hit?
  - What are the coordinates of the hit position?
  - When a ray is reflected/refracted, should the intersection of <u>all</u> objects be considered for the new ray?
  - …
- Performance issue:
  - Naive ray tracing = #pixels x #triangles x #bounces
  - This is really ***slow***, and why ray tracing is so hard

This Scene is Rendered using Ray Tracing (in Real-Time)

- Each statue has more than 33 million triangles
- Naive ray tracing would not work here in any cases
- A clever acceleration structure is needed for sure

Image source: Unreal Engine 5

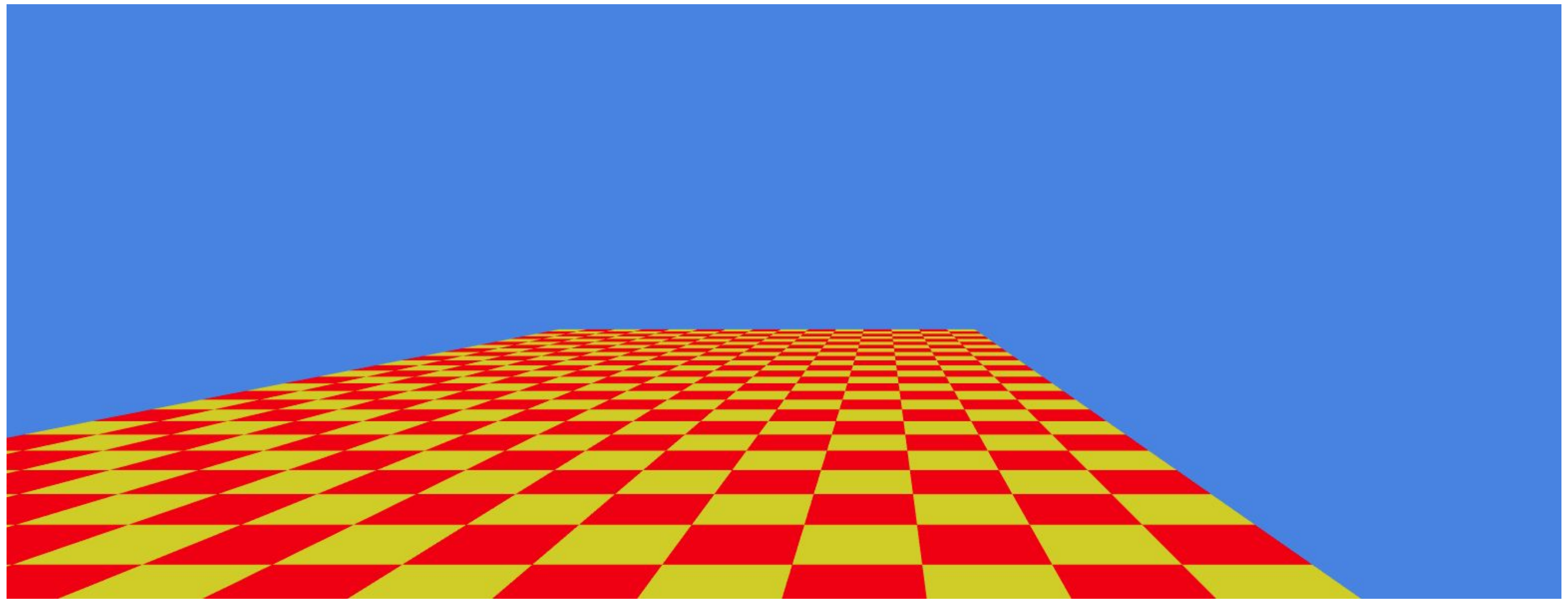
Can we fake it with Rasterization (in Real-Time)?



Task 1 a) Add Checkerboard

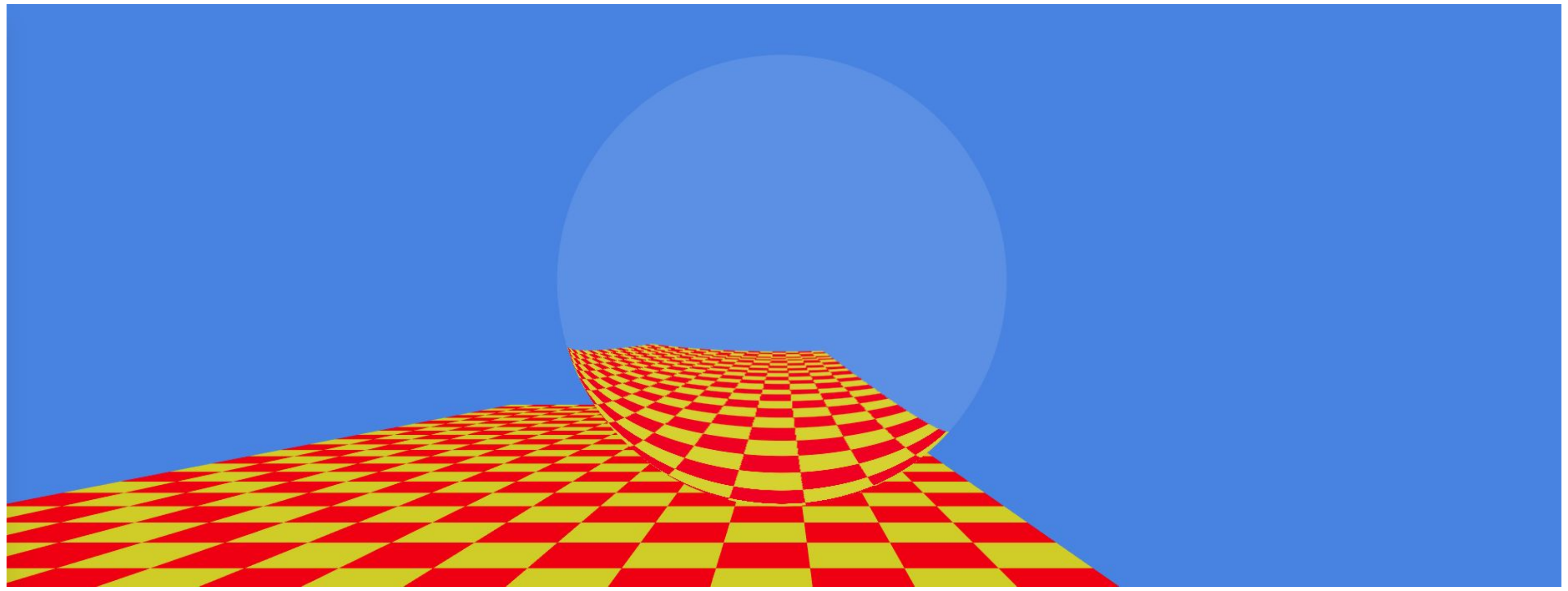
```
constructor() {
  ...
  // TODO: create a checkerboard with red and yellow color
  const g = new PlaneGeometry(
    this.params.plane_width, this.params.plane_height,
    this.params.plane_width, this.params.plane_height
  )
  for (let j = 0; j < this.params.plane_height*2; j+=2) {
    for (let i = 0; i < this.params.plane_width*2; i+=2) {
      g.faces[this.params.plane_height*2*j + i].materialIndex = (i/2+j/2) % 2 === 0 ? 0 : 1
      g.faces[this.params.plane_height*2*j + i+1].materialIndex = (i/2+j/2) % 2 === 0 ? 0 : 1
    }
  }
  this.p = new Mesh(g, [
    new MeshPhongMaterial({color: this.params.color_plane[0]}),
    new MeshPhongMaterial({color: this.params.color_plane[1]})
  ])
  this.p.position.copy(this.params.position.plane)
  this.p.rotateX(-Math.PI/2)
  this.scene.add(this.p)

  // enable shadows
  this.enableShadow()
}
```



Task 1 a) Add Glass Sphere

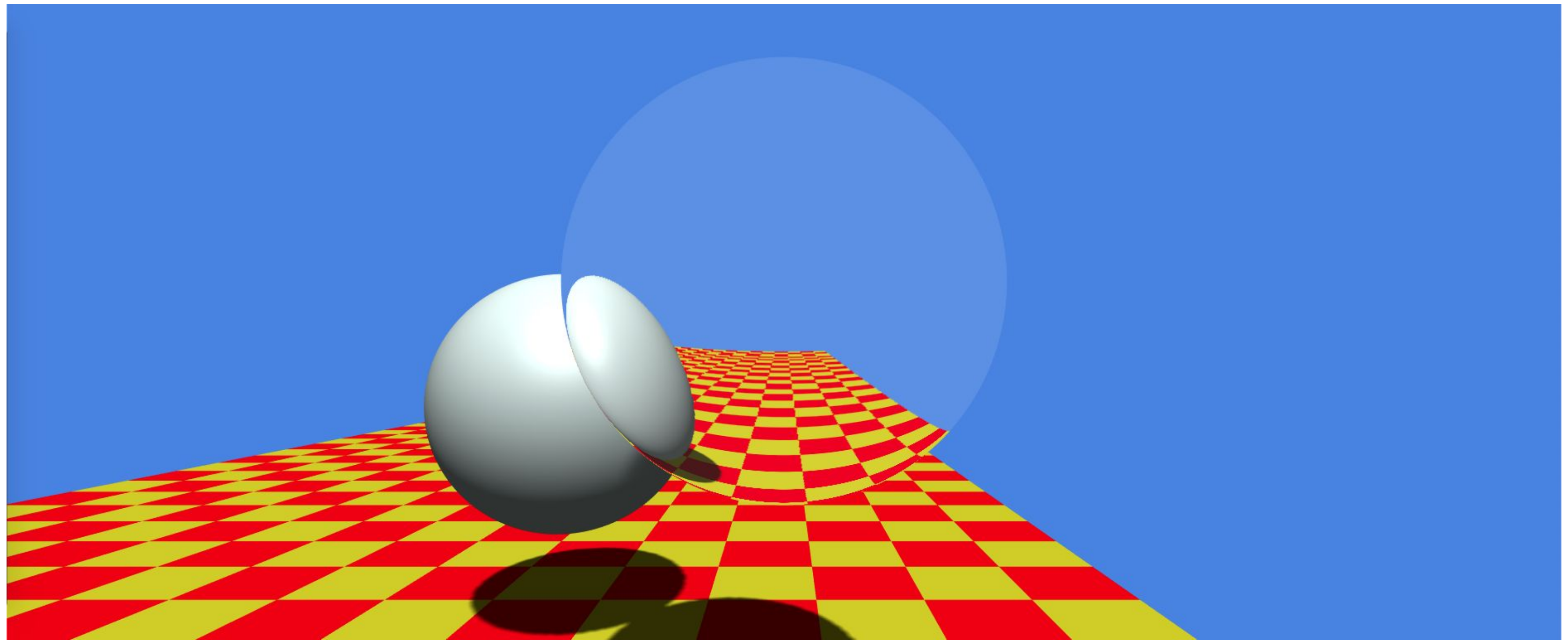
```
constructor() {
  ...
  // TODO: create a glass sphere based on MeshBasicMaterial.
  this.refractionCamera = new CubeCamera(0.1, 5000, 512)
  this.refractionCamera.renderTarget.mapping = CubeRefractionMapping
  this.scene.add(this.refractionCamera)
  this.s1 = new Mesh(
    new SphereGeometry(this.params.radius, 100, 100),
    new MeshBasicMaterial({
      color: 0xffffff,
      envMap: this.refractionCamera.renderTarget,
      side: BackSide,
      refractionRatio: this.params.refractionRatio,
      reflectivity: this.params.reflectivity
    }),
  )
  this.s1.position.copy(this.params.position.right)
  this.refractionCamera.position.copy(this.s1.position)
  this.scene.add(this.s1)
  ...
}
update() {
  // TODO: implement update if you needed (yes we need it).
  this.s1.visible = false
  this.refractionCamera.update(this.renderer, this.scene)
  this.s1.visible = true
}
```



Task 1 a) Add Metal Sphere and Shadows

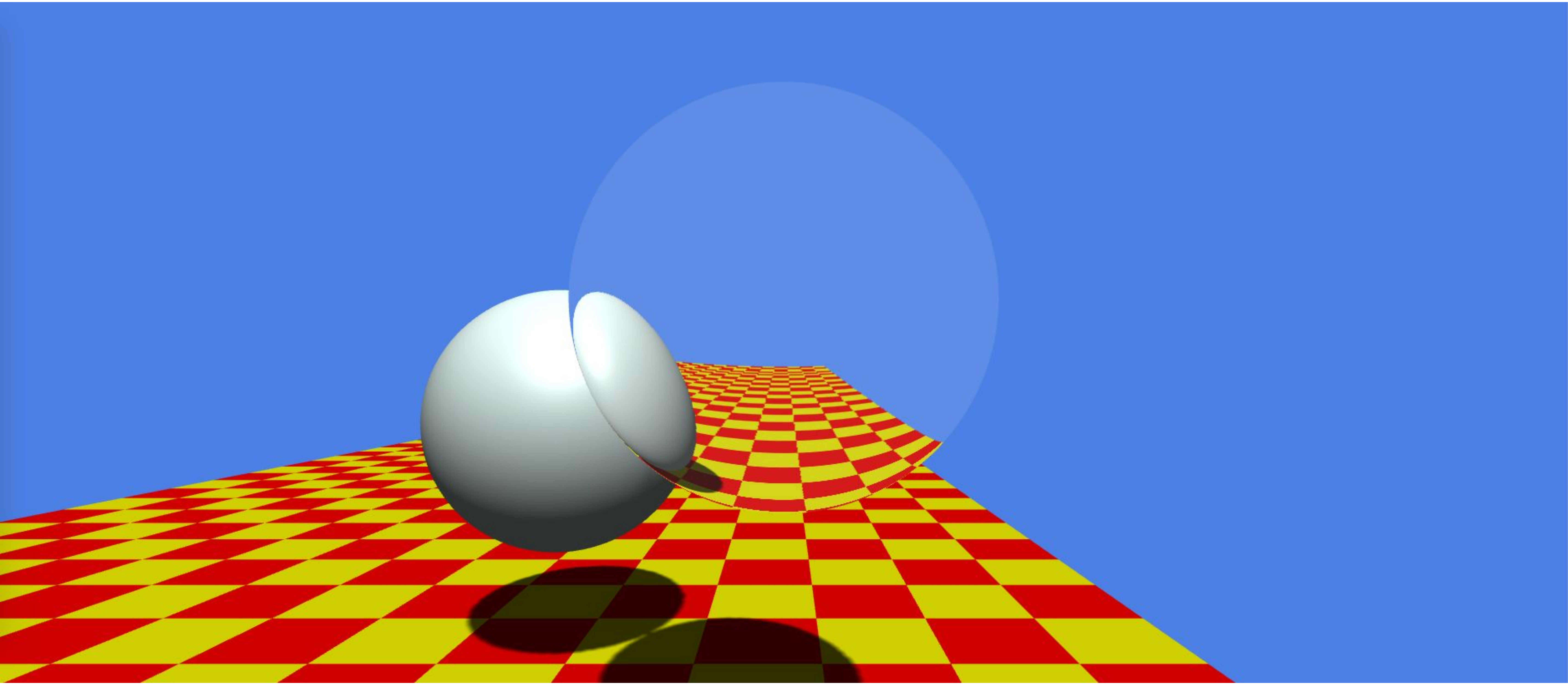
```
constructor() {
  ...
  // TODO: create a metal sphere based on phong material
  this.s2 = new Mesh(
    new SphereGeometry(this.params.radius, 100, 100),
    new MeshPhongMaterial({color: this.params.color_sphere, side: FrontSide}),
  )
  this.s2.position.copy(this.params.position.left)
  this.scene.add(this.s2)
  ...
}

enableShadow() {
  // TODO: enable shadows for objects you have created
  this.renderer.shadowMap.enabled = true
  this.renderer.shadowMap.type = PCFSoftShadowMap
  this.s1.castShadow = true
  this.s1.receiveShadow = true
  this.s2.castShadow = true
  this.s2.receiveShadow = true
  this.p.receiveShadow = true
}
```



A Fake Whitted-style

Live Demo: https://www.medien.ifi.lmu.de/lehre/ss20/cg1/demo/7-illumination/whitted/index.html



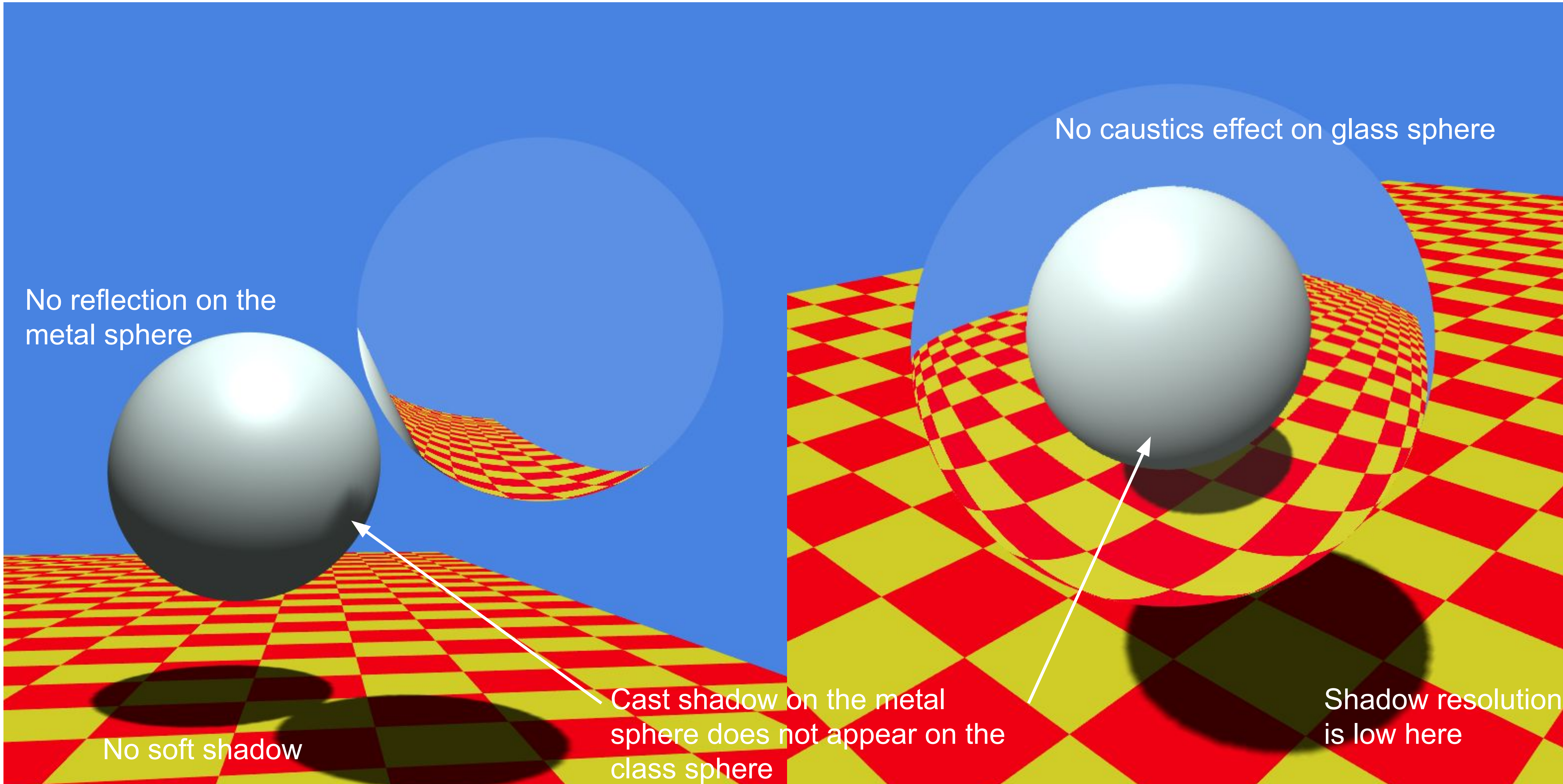
Other Possible Solutions

The demonstrated solution is not perfect, you could also come up with other solutions using:

- Customized shaders
- Texture mapping
- …

Be more creative :)

Task 1 b) What went wrong here?



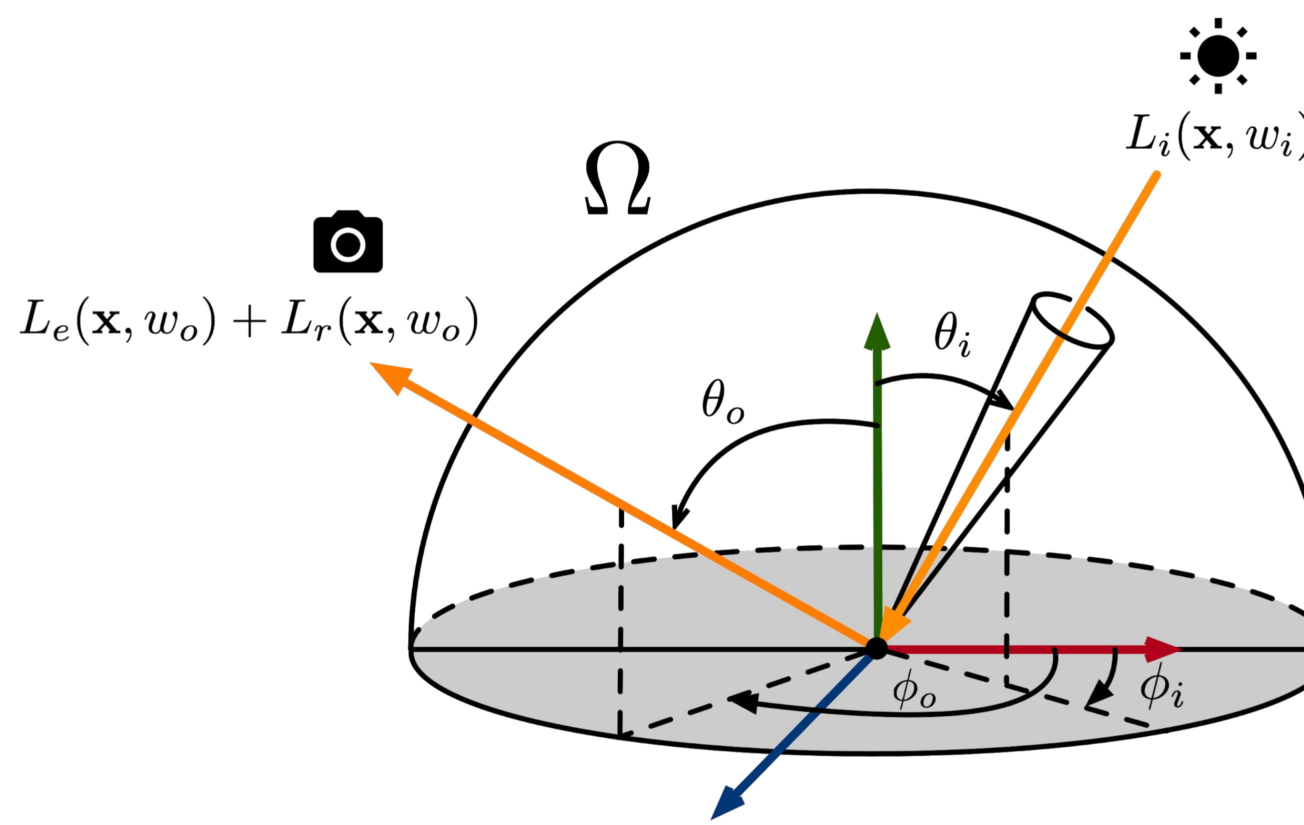
Task 1 c) More physically-based Effects?

- Absorption
- Scattering
- Polarization
- Diffraction
- Interference
- …

None of them are possible with Whitted-style ray tracing.

Revisit: The Rendering Equation (1986)

$$L_o(\mathbf{x}, w_o) = L_e(\mathbf{x}, w_o) + L_r(\mathbf{x}, w_o) = L_e(\mathbf{x}, w_o) + \int_\Omega f_r(\mathbf{x}, w_i, w_o) L_i(\mathbf{x}, w_i) \cos\theta_i \mathrm{d}w_i$$



James T. Kajiya. 1986. The rendering equation. In Proceedings of the 13th annual conference on Computer graphics and interactive techniques (SIGGRAPH '86). Association for Computing Machinery, New York, NY, USA, 143–150. DOI:https://doi.org/10.1145/15922.15902

Task 1 d) Whitted-style is "wrong"

The rendering equation integrates the radiance from all incoming directions (hemisphere) but the whitted style ray tracing does not consider other indirect effects (single ray path).

The rendering equation is "correct", at least more correct than Whitted-style.

But how to solve it?

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- **Monte Carlo Ray Tracing**
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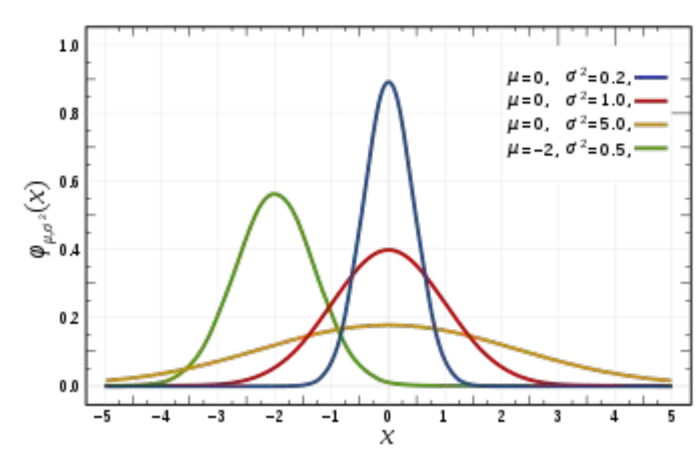
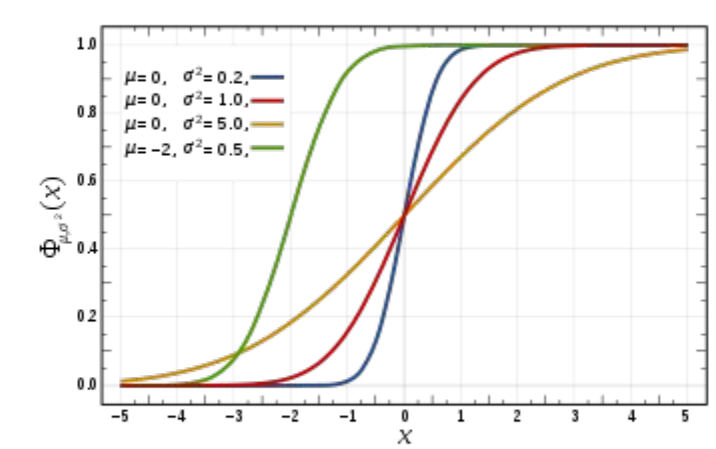
Revisit: Probability and Statistics

- (Continuous) random variables $X \sim p(x)$
- Probability $P(a \leq X \leq b) = \int_a^b p(x)\mathrm{d}x$
- Cumulative distribution function (CDF) $P(X \leq x) = \int_{-\infty}^{x} p(t)\mathrm{d}t$
- Probability density function (PDF) $p(x)$, e.g. Gaussian
- Expected value $E(x) = \int_{-\infty}^{\infty} xp(x)\mathrm{d}x$



PDF: Gaussian



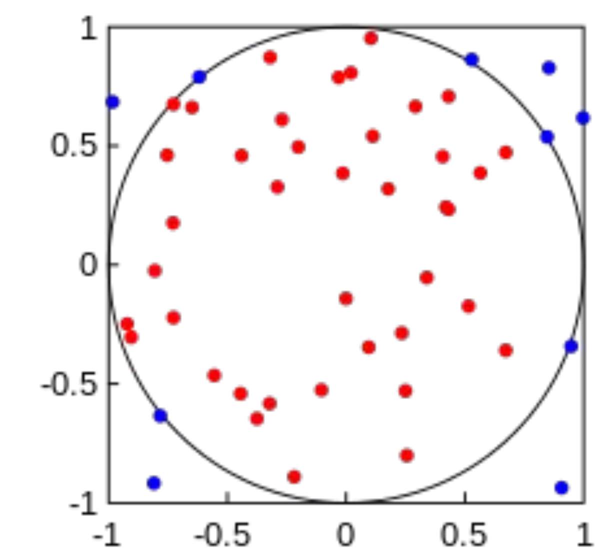
CDF: Gaussian

https://en.wikipedia.org/wiki/Normal_distribution

Monte Carlo Integration

It can be too hard to solve a definite integration analytically. The Monte Carlo Integration is a method to estimate the definite integral of a function by averaging random samples of the function's value.

The rendering equation can be solved using the Monte Carlo integration:



https://en.wikipedia.org/wiki/Monte_Carlo_integration

$$\int_a^b f(x)\mathrm{d}x = \frac{1}{N}\sum_{k=1}^{N}\frac{f(X_k)}{p(X_k)}, X_k \sim p(x)$$

The rendering equation using Monte Carlo integration is:

$$L_o(x, w_o) = L_e(x, w_o) + \frac{1}{N}\sum_{k=1}^{N}\frac{L_{i,k}(x, w_{i,k})f_r(x, w_{i,k}, w_{o,k})\cos\theta_{i,k}}{p(w_{i,k})}$$

Task 2 a) The Rendering Equation with Lambertian BRDF

In Assignment 6 - Task 3 a), we derived the BRDF $f_r = \frac{1}{\pi}$ for a Lambertian surface/material.

Note that the Lambertian material only takes into account diffuse incoming radiance, and doesn't absorb or emit radiance, i.e. $L_e(x, w_o) = 0$

So the rendering equation using Monte Carlo integration is:

$$L_o(x, w_o) = \frac{1}{N} \sum_{k=1}^{N} \frac{L_{i,k}(x, w_{i,k}) \cos \theta_{i,k}}{\pi p(w_{i,k})}$$

Task 2 b) Uniform Sampling on Hemisphere

Uniform sampling means the density function is a constant, thus $p(w_{i,k}) = c$

Due to the definition of the probability density function (PDF), we have:

$$\int_\Omega p(w_{i,k}) \mathrm{d}w_{i,k} = c \int_\Omega \mathrm{d}w_{i,k} = 1$$

The definite integral is on a hemisphere, with the spherical coordinates:

$$\begin{aligned}
\int_\Omega \mathrm{d}w_{i,k} &= \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \mathrm{d}\theta \sin\theta \mathrm{d}\phi \\
&= \int_0^{\pi/2} \sin\theta \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \\
&= (-\cos\frac{\pi}{2} + \cos 0)(2\pi - 0) = 2\pi
\end{aligned}$$

Therefore $p_{\mathrm{hemi}}(w) = p(w_{i,k}) = \dfrac{1}{2\pi}$ (i.e. 1 / surface area)

Solving The Rendering Equation (The Easy/Naive Case)

The rendering equation on a Lambertian surface, with uniform sampling on a hemisphere can be simplified to:

$$L_o(\mathbf{x}, w_o) = L_e(\mathbf{x}, w_o) + \int_\Omega f_r(\mathbf{x}, w_i, w_o) L_i(\mathbf{x}, w_i) \cos\theta_i \mathrm{d}w_i = \frac{2}{N} \sum_{k=1}^{N} L_{i,k}(x, w_{i,k}) \cos\theta_{i,k}$$

A simple Monte Carlo estimation for the rendering equation (pseudocode):

```
// x is shading point, wo is the outgoing ray
shade(x, wo, bounces) {
    Lo = 0
    if bounces == 0 { return hit light ? light_emission : 0 } // termination condition of recursive function call
    for each randomly sample N directions wi {
        trace a ray
        if ray hit the light: Lo += light_emission * cosine_theta
        if ray hit object at q: Lo += shade(q, -wi, bounces-1) * cosine_theta // f_r = 1/pi, pdf(wi) = 1\2pi
    }
    return 2*Lo/N
}
```

Recursive depth == #bounces, thus #rays = $N^{\#bounces}$ ⇒ explode!

When N=1: Path tracing, just a single ray

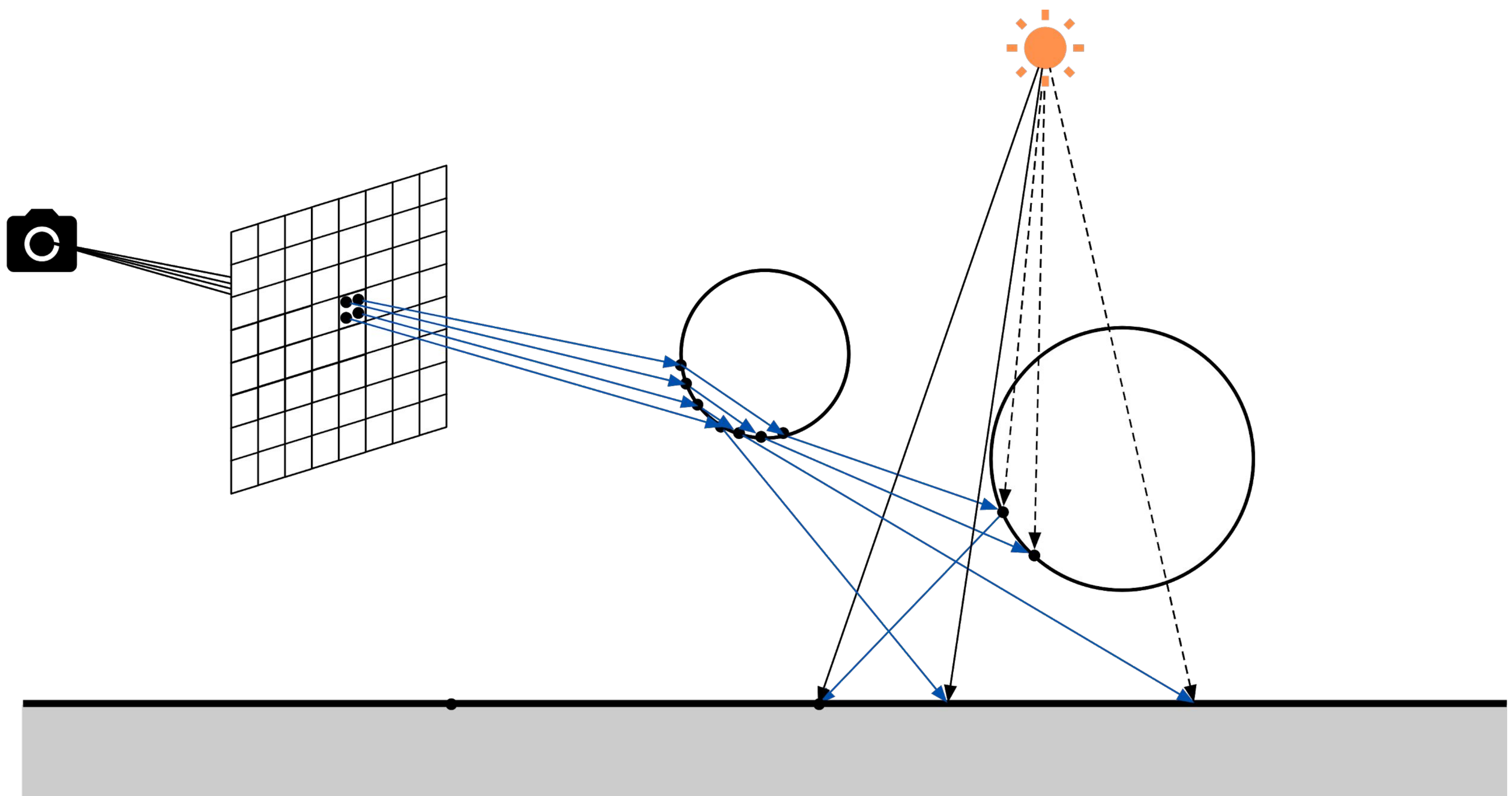
When N>1: Distributed ray tracing, tracing exploded #rays

Global Illumination via Path Tracing

Sampling multiple random directions can cause #ray to explode

But we can shoot multiple ray paths from a pixel to gain linear complexity

i.e. multiple *samples per pixel* (spp)



Global Illumination via Path Tracing (cont.)

```
// x is shading point, wo is the outgoing ray
shade(x, wo, bounces) {
    Lo = 0
    if bounces == 0 { return hit light ? light_emission : 0 }
    randomly sample ONE direction wi
    trace a ray
    if ray hit the light: Lo += light_emission * cosine_theta
    if ray hit object at q: Lo += shade(q, -wi, bounces-1) * cosine_theta // f_r = 1/pi, pdf(wi) = 1\2pi
    return 2*Lo/N
}
```

What if we running out of the #bounces and could not hit the light?

The whole recursion will get 0 ⇒ **inefficient. A lot of path are "wasted"**

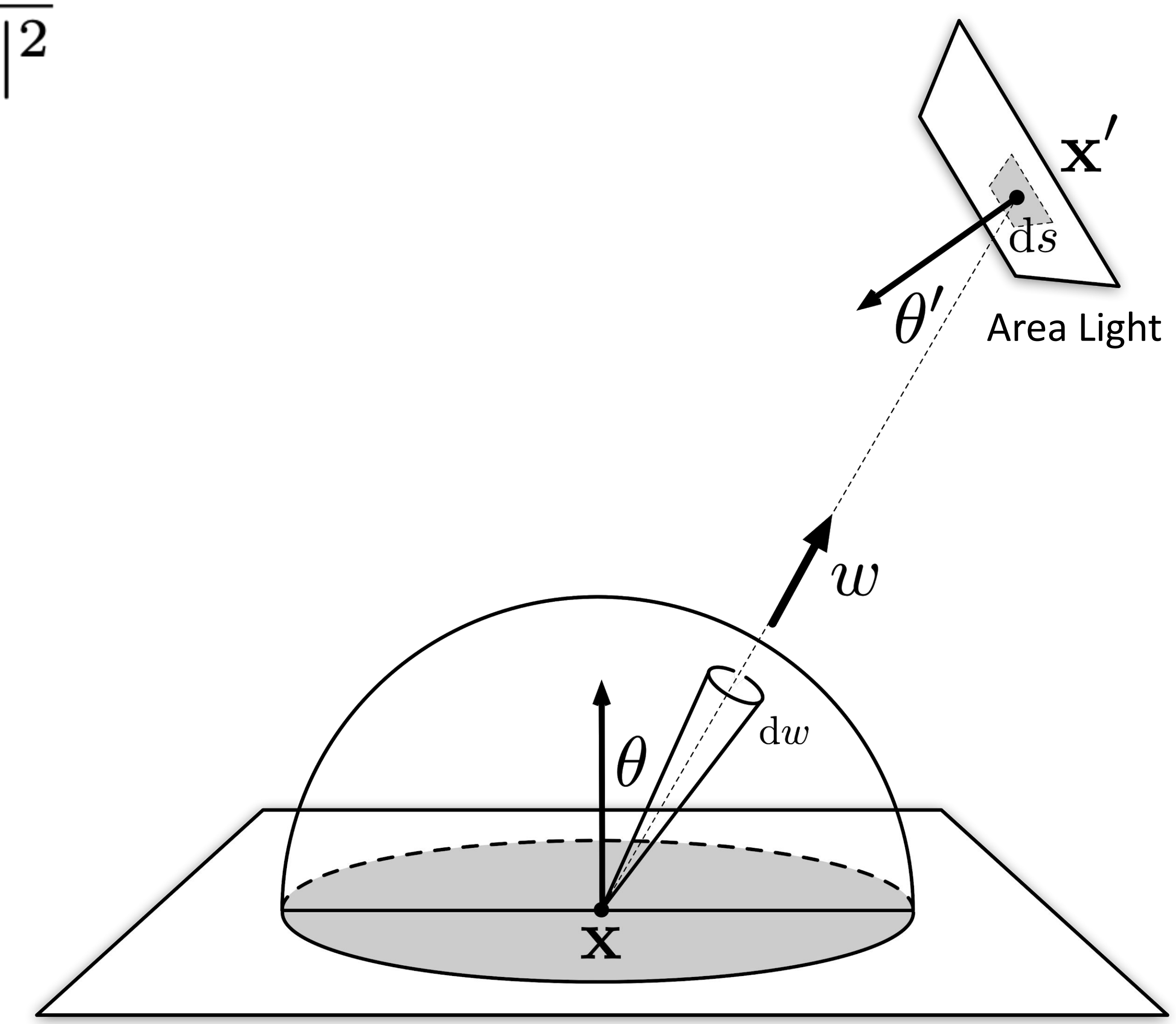
What can we do about this?

Task 2 c) Sampling Area Light (Direct Illumination)

Recall the definition of a solid angle (it is the projected area on the unit sphere)

We can express d***w*** by d***s*** immediately using the definition:

$$\mathrm{d}w = \frac{\cos\theta' \mathrm{d}s}{|\mathbf{x} - \mathbf{x}'|^2}$$



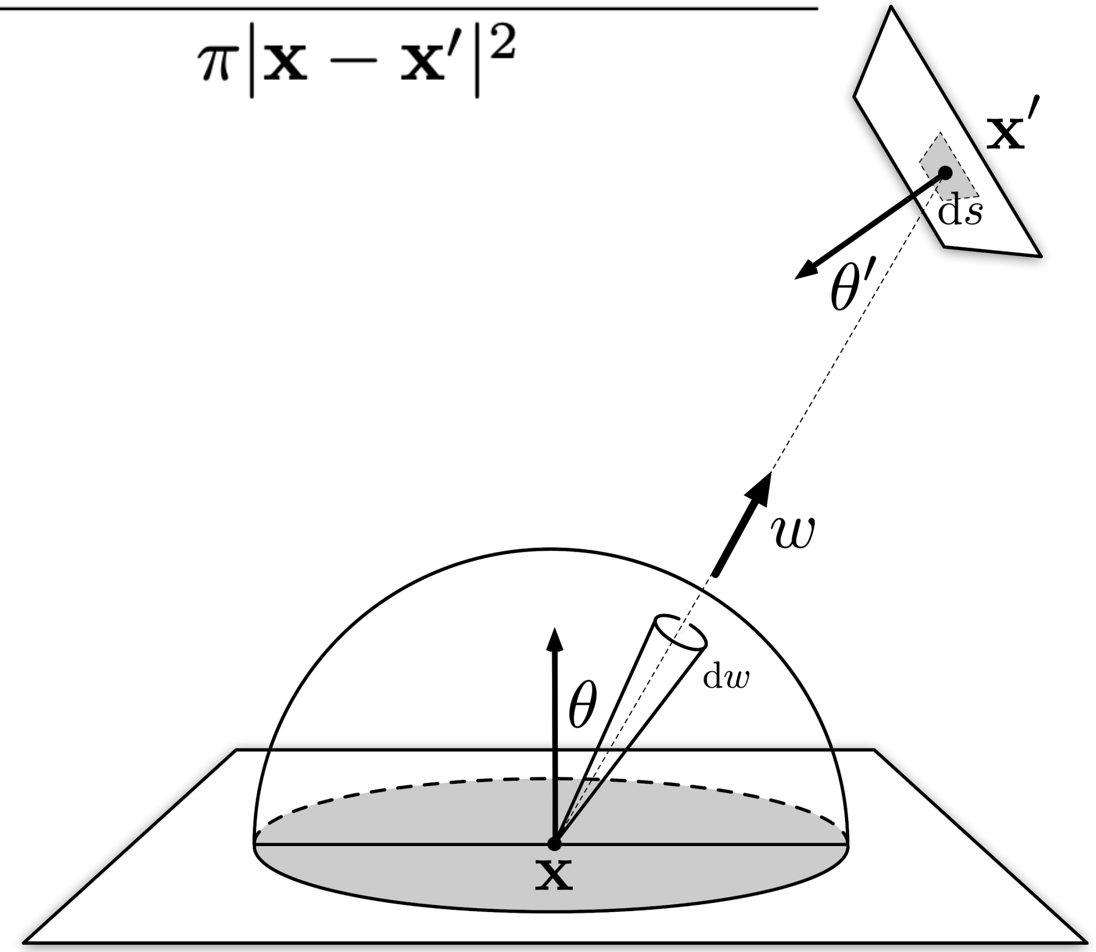
The Rendering Equation by Sampling Light Source

In this case, the PDF is *1/S* . The rendering equation is

$$L_o(\mathbf{x}, w_o) = L_e(\mathbf{x}, w_o) + \int_S f_r(\mathbf{x}, w_i, w_o) L_i(\mathbf{x}, w_i) \frac{\cos\theta_i \cos\theta'}{|\mathbf{x} - \mathbf{x}'|^2} \mathrm{d}s$$

The corresponding Monte Carlo solution that samples an area light:

$$L_o(x, w_o) = \frac{1}{N} \sum_{k=1}^{N} \frac{L_{i,k}(x, w_{i,k}) \cos\theta_{i,k} \cos\theta'_{i,k}}{\pi \frac{1}{S} |\mathbf{x} - \mathbf{x}'|^2} = \frac{1}{N} \sum_{k=1}^{N} \frac{L_{i,k}(x, w_{i,k}) \cos\theta_{i,k} \cos\theta'_{i,k} S}{\pi |\mathbf{x} - \mathbf{x}'|^2}$$



Task 2 d) Implementing A Simple Path Tracer

In the lecture, you learned that WebGL 2.0 does not support recursion, thus we need to turn the accumulation to a non-recursive version. The pseudocode from Assignment 7 already tells you how to do it:

```
shade(wo) {
   Li = 0
   Lo = 0
   for i = 0; i < bounces; i++ {
       if wo not hit the world {
           return Lo
       }
       if wo hit light source {
           return Lo
       }
       Li = Li * hitted material color
       if light is not blocked in the middle {
           Lo += radiance at hit position // use the rendering equation
       }
       wo = randomly sample one direction
   }
   return Lo
}
```

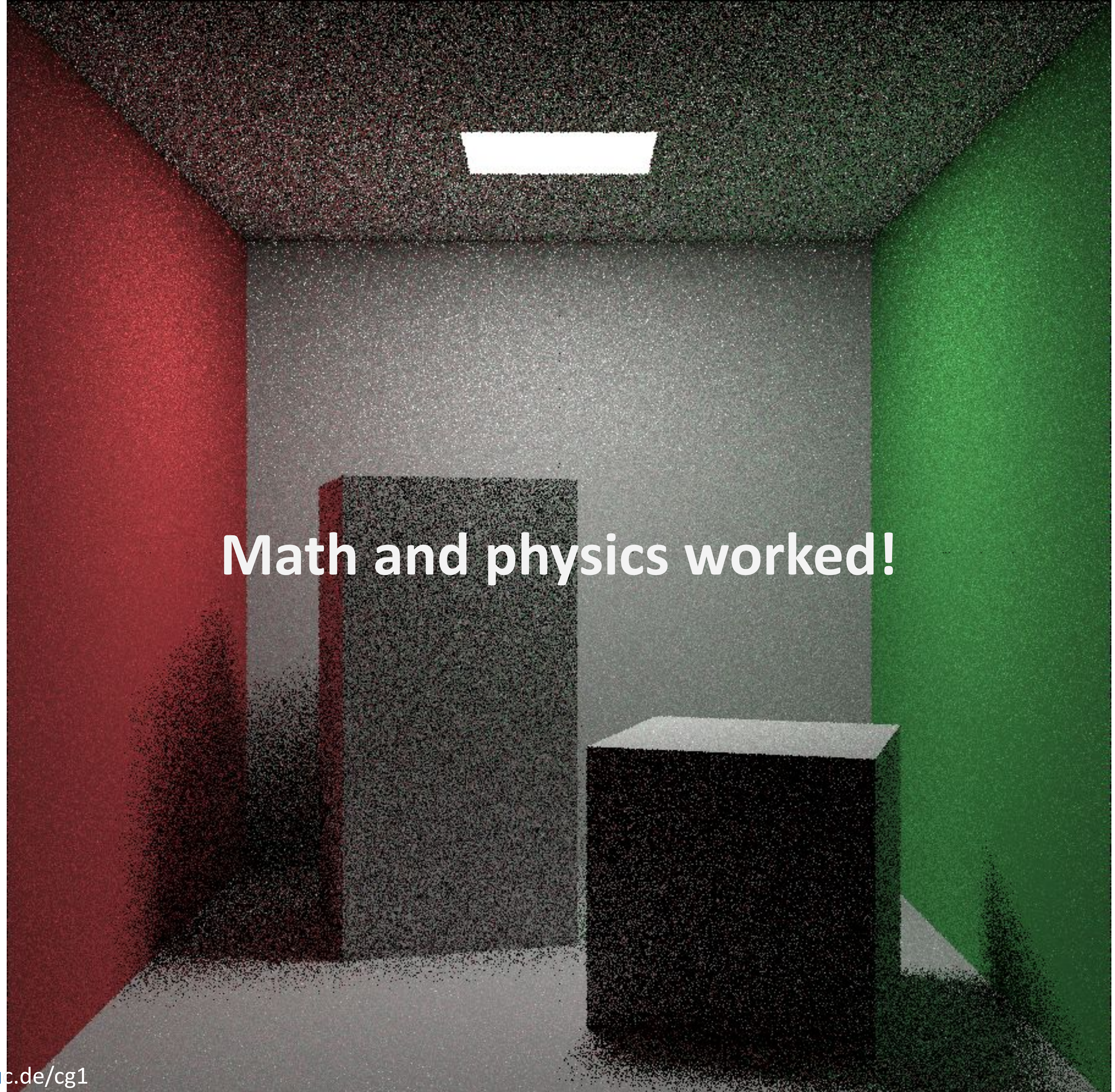
$$\frac{L_{i,k}(x, w_{i,k}) \cos\theta_{i,k} \cos\theta'_{i,k} S}{\pi|\mathbf{x} - \mathbf{x}'|^2}$$

Task 2 d) Implementing A Simple Path Tracer (cont.)

- 14 lines of code (can be shorter by remove intermediate variables)
- Indeed difficult to understand from beginning to the end
- Even not easy to debug
- But you can get a huge sense of accomplishment for sure if you did it right

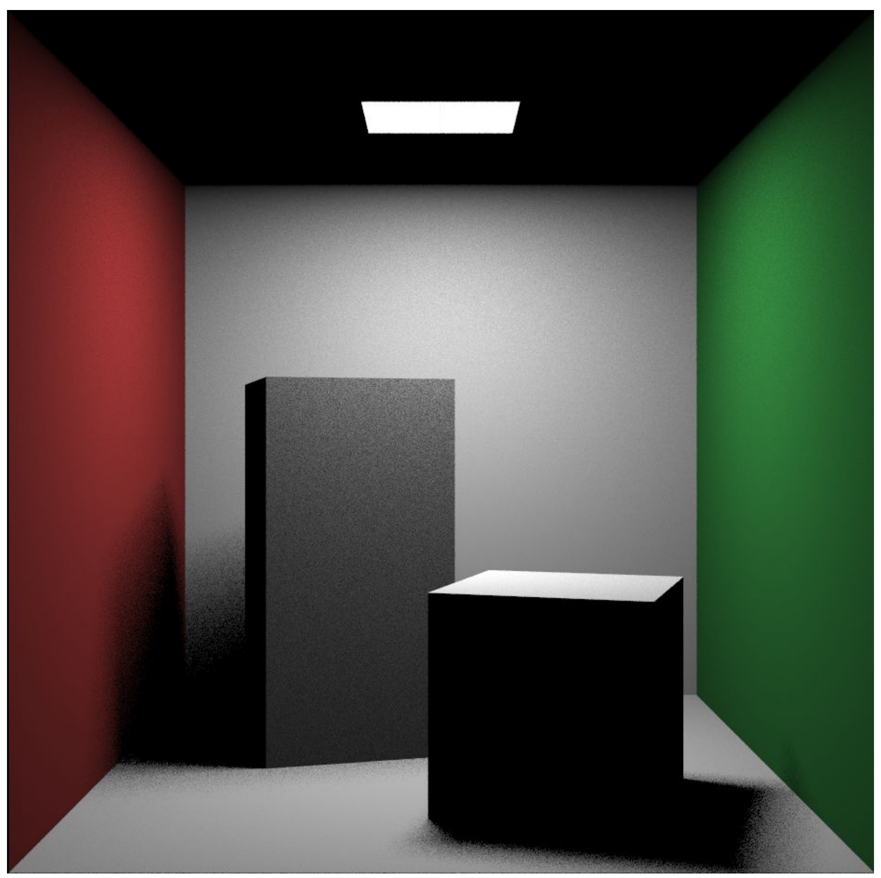
```
vec3 shade(in ray r) {
   vec3 Li = vec3(0); vec3 Lo = vec3(0); hit_record hit;
   float pdf_light = 1.0 / area_light_surface; // 1/S
   float f_r = 1.0 / pi;                        // lambertian
   // TODO: implement path tracing, return the emitted color of the given ray.
   for (int i = 0; i < 1+bounces; i++) {
       if (!world_hit(r, hit)) { return Lo; }
       if (hit.mat.type == light_source) { return i == 0 ? vec3(light.color) : Lo; }
       vec3 Lpos = area_light.center + area_light.dimension * (2.*random3()-1.);
       vec3 l = Lpos - hit.p;
       float distance_sequre = dot(l, l);                   // |x-x'|^2
       l = normalize(l);
       ray shadow_ray = ray(hit.p, l);
       Li = i == 0 ? hit.mat.color : Li * hit.mat.color;
       float cosine_theta1 = dot(hit.normal, l);            // cosθ
       float cosine_theta2 = dot(area_light_normal, -l);    // cosθ'
       if (cosine_theta1 > 0.0 && cosine_theta2 > 0.0 && !shadow_hit(shadow_ray)) { // direct illumination
           Lo += Li * light.color * f_r * cosine_theta1 * cosine_theta2 / distance_sequre / pdf_light;
       }
       r = ray(hit.p, sample_wi(hit.normal)); // randomly sample one direction, for the next trace
   }
   return Lo;
}
vec3 ray_generation(camera c, vec2 frag_cood) { // generating rays
   vec3 Lo = vec3(0);
   for (int i = 0; i < spp; i++) {
       ray r = ...     // omitted here, see code skeleton
       Lo += shade(r);
   }
   return Lo / float(spp);
}
```

$$\boxed{\frac{1}{N}\sum_{k=1}^{N}}\boxed{\frac{L_{i,k}(x, w_{i,k})\cos\theta_{i,k}\cos\theta'_{i,k}S}{\pi|\mathbf{x}-\mathbf{x}'|^2}}$$

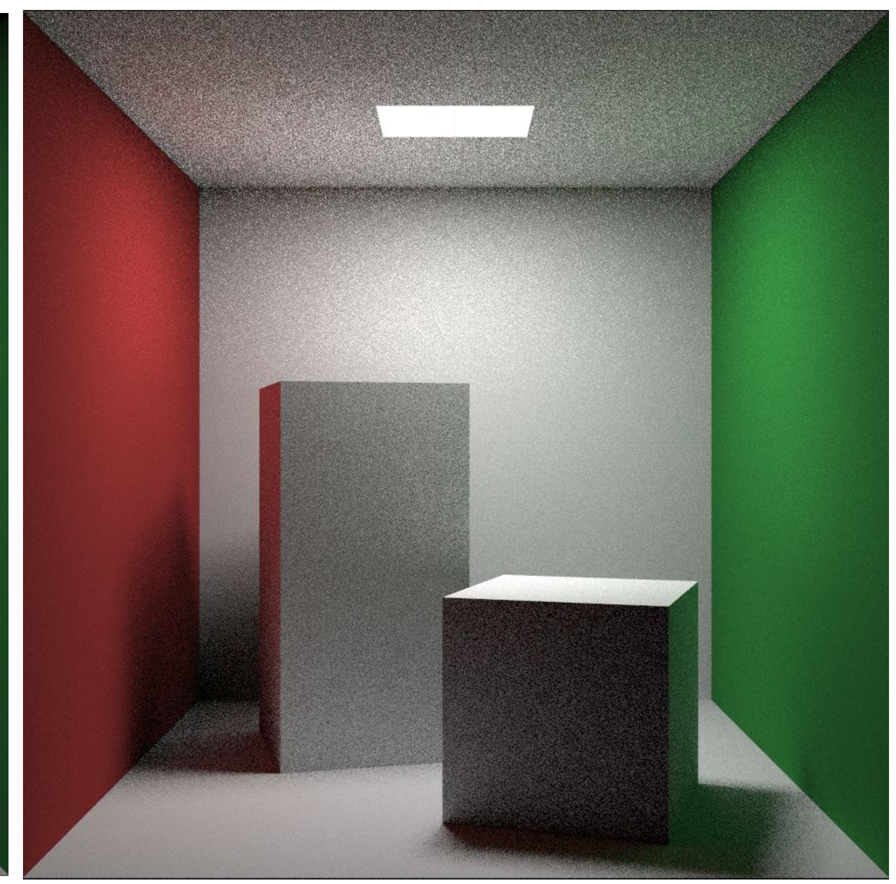

Math and physics worked!

Task 2 e) Changing Light Bounces

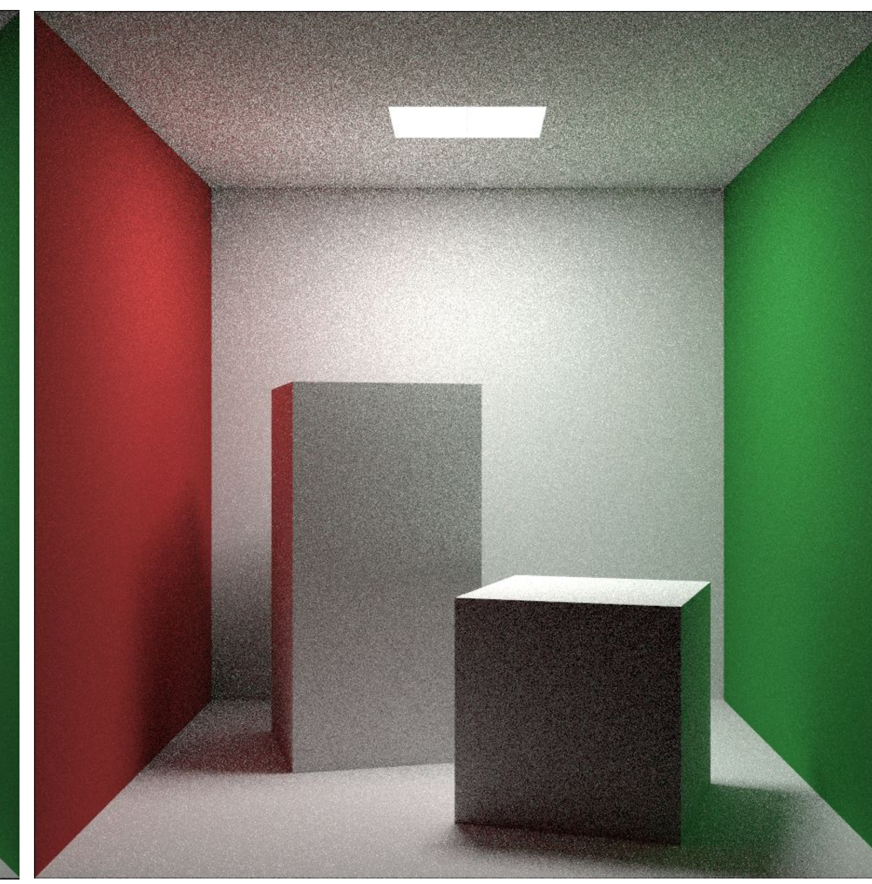
- Increasing light bounces introduces more indirect illumination to non emission materials
- Infinite light bounces won't make the scene become pure white as how real world light behaves (or energy conservation: reflected radiance ≤ incoming radiance)



no light bounce, 20 spp



1 light bounce, 20 spp



2 light bounce, 20 spp

But how to decide the number of light bounces?

Biased vs. Unbiased Monte Carlo Estimator

- Unbiased: No systematic error, or the *expected value* of the Monte Carlo estimator is equal to the definite integral
- Biased: otherwise

Without infinite light bounces, our calculated color is always biased.

What can we do about this?

Solution: Russian Roulette (Using Bernoulli Distribution)

Assume we don't set a limit to the number of light bounces, instead of manually giving a probability *p* (0 < *p* < 1)

- With a probability p, we keep shooting (reflect) an accumulate ray at the Monte Carlo integration, the accumulate radiance is divided by *p*: **Li / p**
- With a probability *1-p*, we terminate bouncing the ray ⇒ returns radiance: **0**

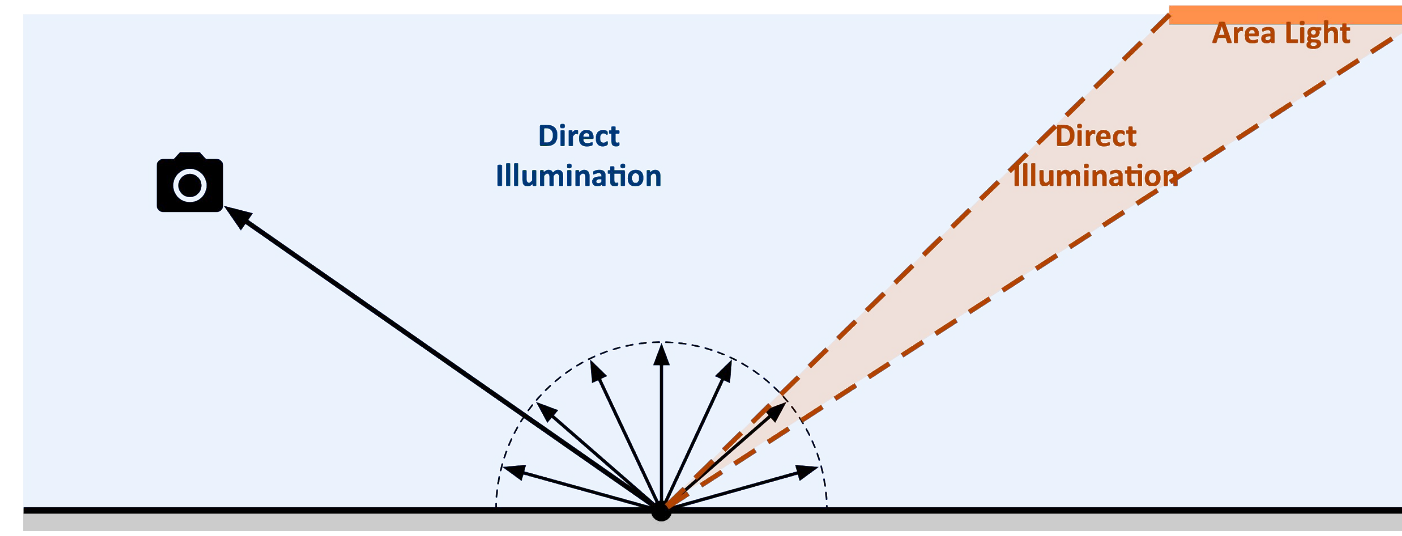
The expected value (i.e. Bernoulli 0-1 distribution):

$$E = p \cdot \sum_i \frac{L_i}{p} + (1-p) \cdot 0 = \sum_i L_i = L_o$$

Now we don't rely on the number of bounces and have an unbiased estimation

(even doesn't depends on the probability you have chosen anymore)

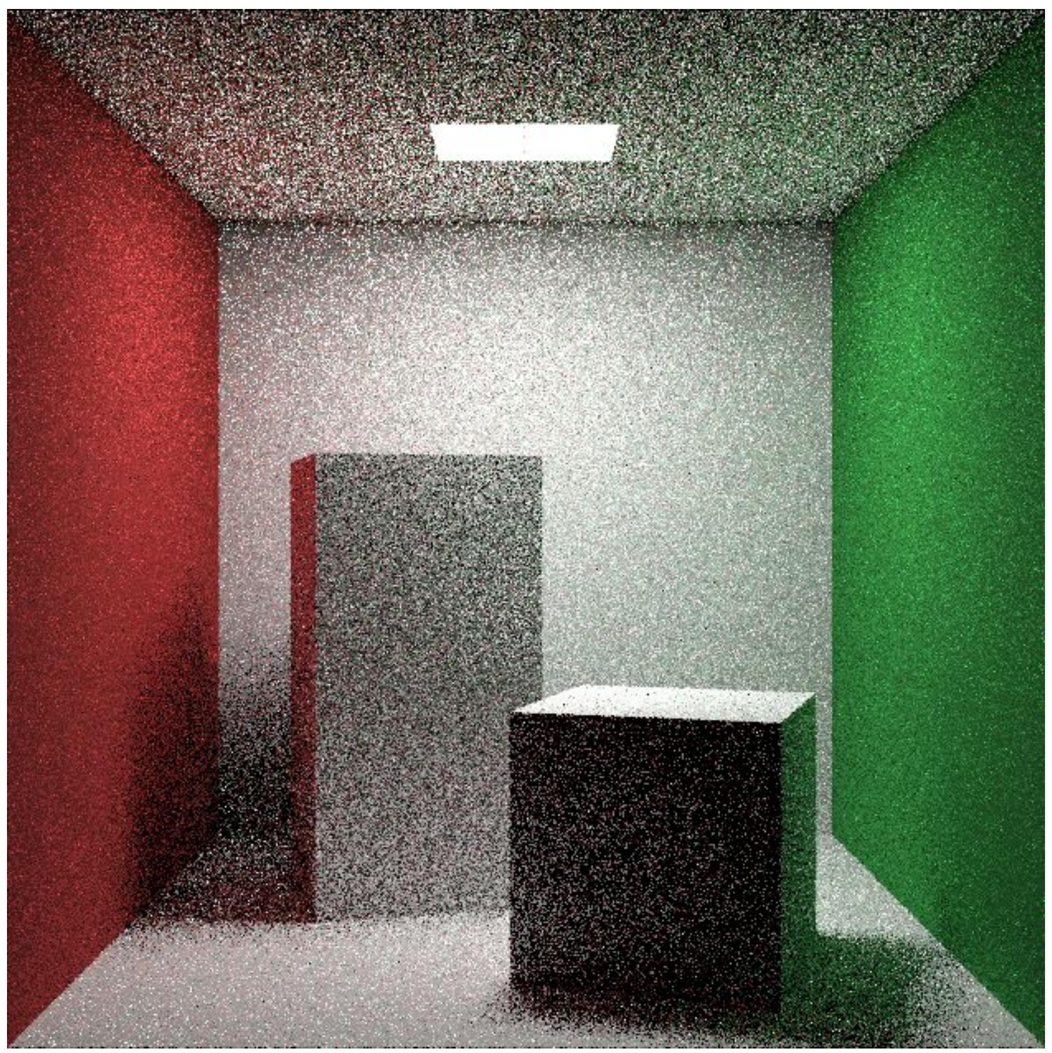
Path Tracing (unbiased complete version, pseudocode)



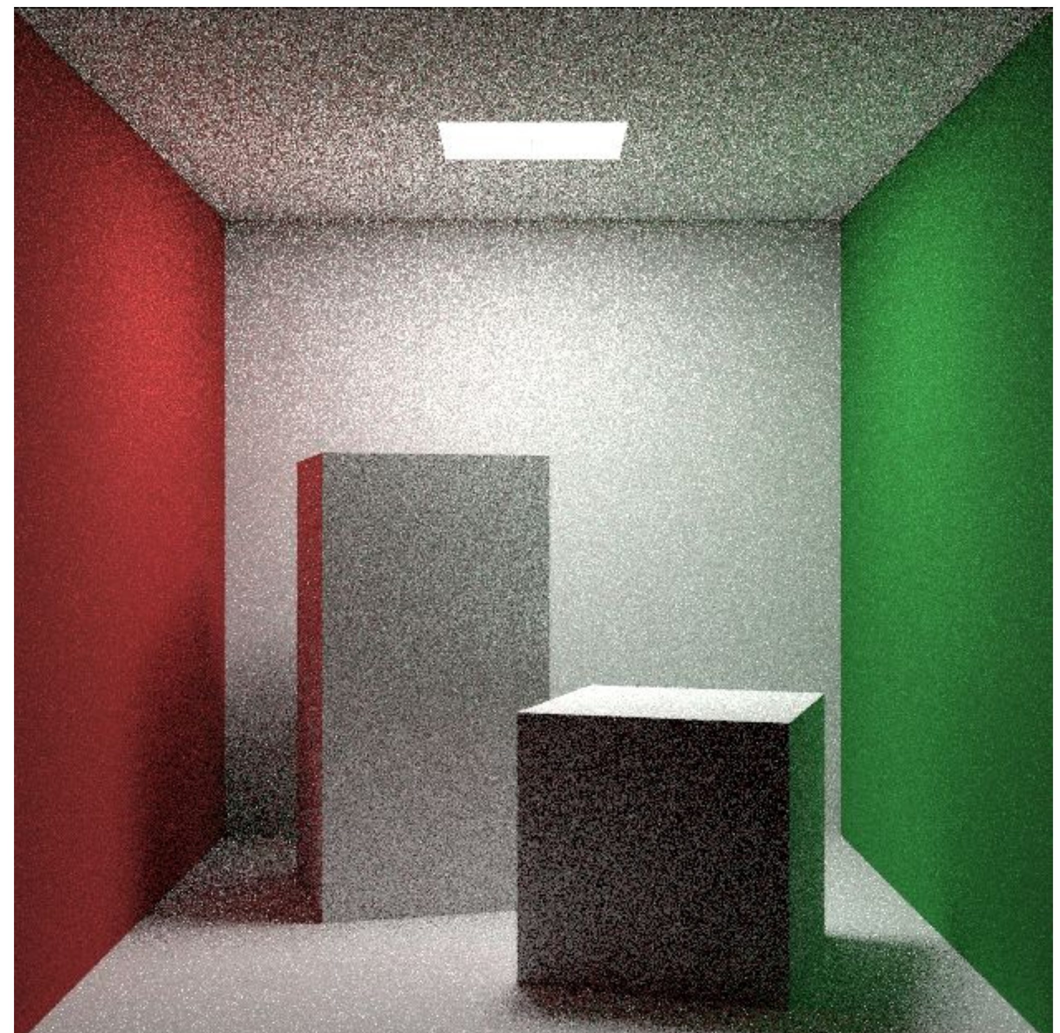

```
// x is shading point, wo is the outgoing ray
shade(x, wo) {
    // direct illumination, contribution from the light source
    Ldir = 0
    ray = x - x' // x' is from light source
    if ray is not blocked in the middle {
        Ldir = light_color * f_r * cosθ * cosθ' / |x'-x|^2 / pdf_light // integrate over light source
    }

    // indirect illumination, contribution from non-emitting materials
    Lindir = 0
    if random() < p_rr { // p_rr is the given russian roulette probability
        wi = sample(wo, n) // hemisphere sampling ONE incoming direction for the next ray
        if wi hit non emitting object at q {
            Lindir = shade(q, -wi) * f_r * cosθ / pdf_hemi / p_rr // task 2 b): pdf_hemi = 1/2pi
        }
    }
    return Ldir + Lindir
}
```

Task 2 f) Changing spp

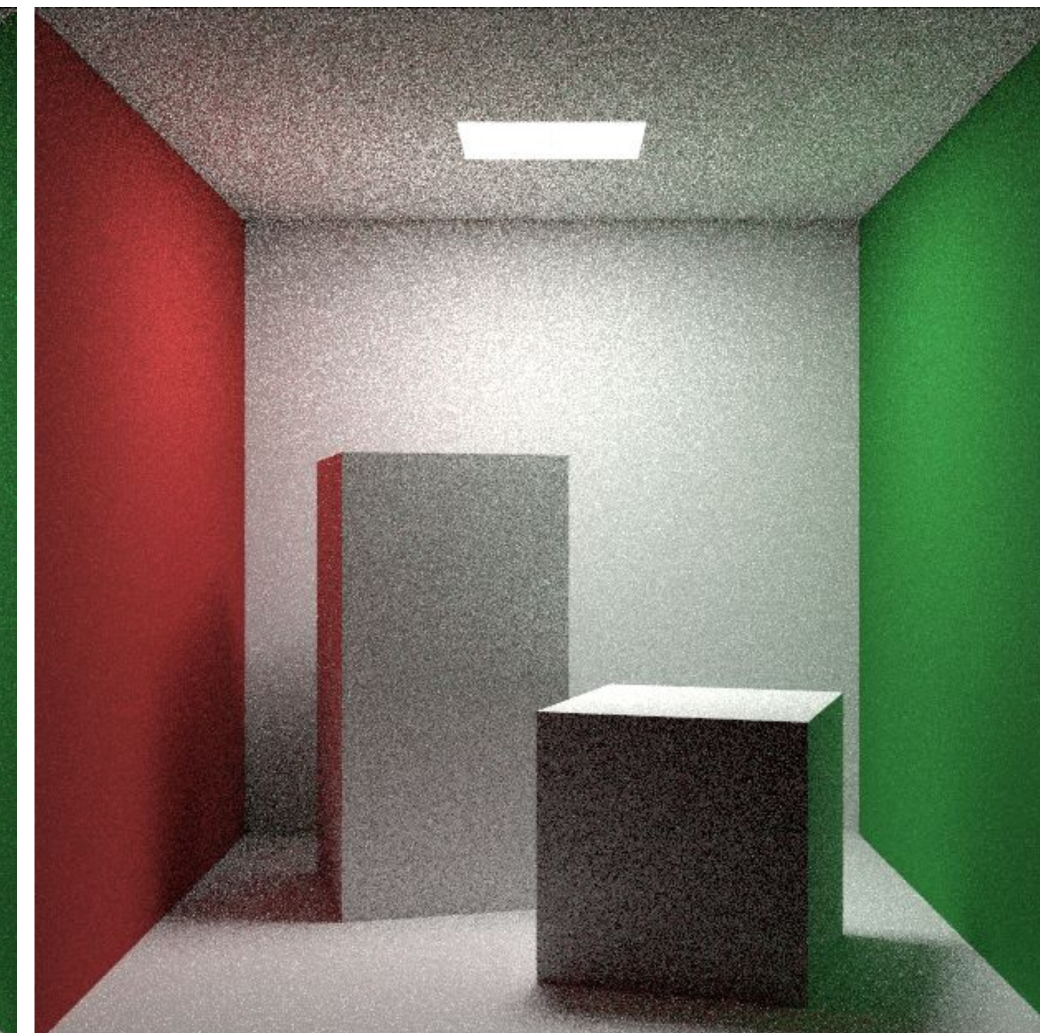
- Increasing spp can reduce the sampling noise
- Because of the law of large numbers, we need infinite samples per pixel eventually



2 light bounce, 1 spp



2 light bounce, 4 spp

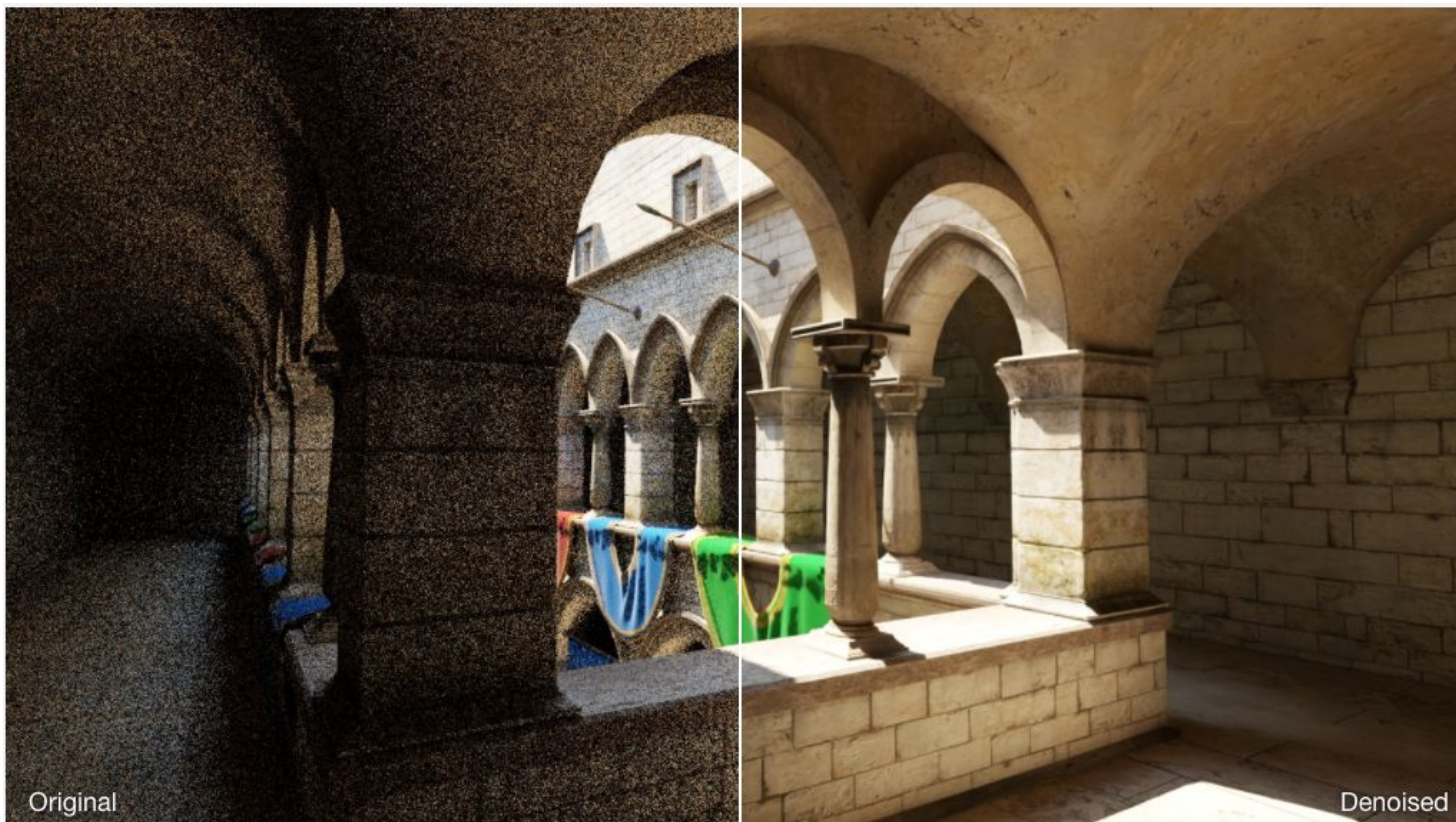


2 light bounce, 16 spp

But what can we do about it?

Denoising!

- An appropriate denoising approach can reduce the samples per pixel that we need
- Ideally, we want a denoised image that rendered with 1 spp (why?)

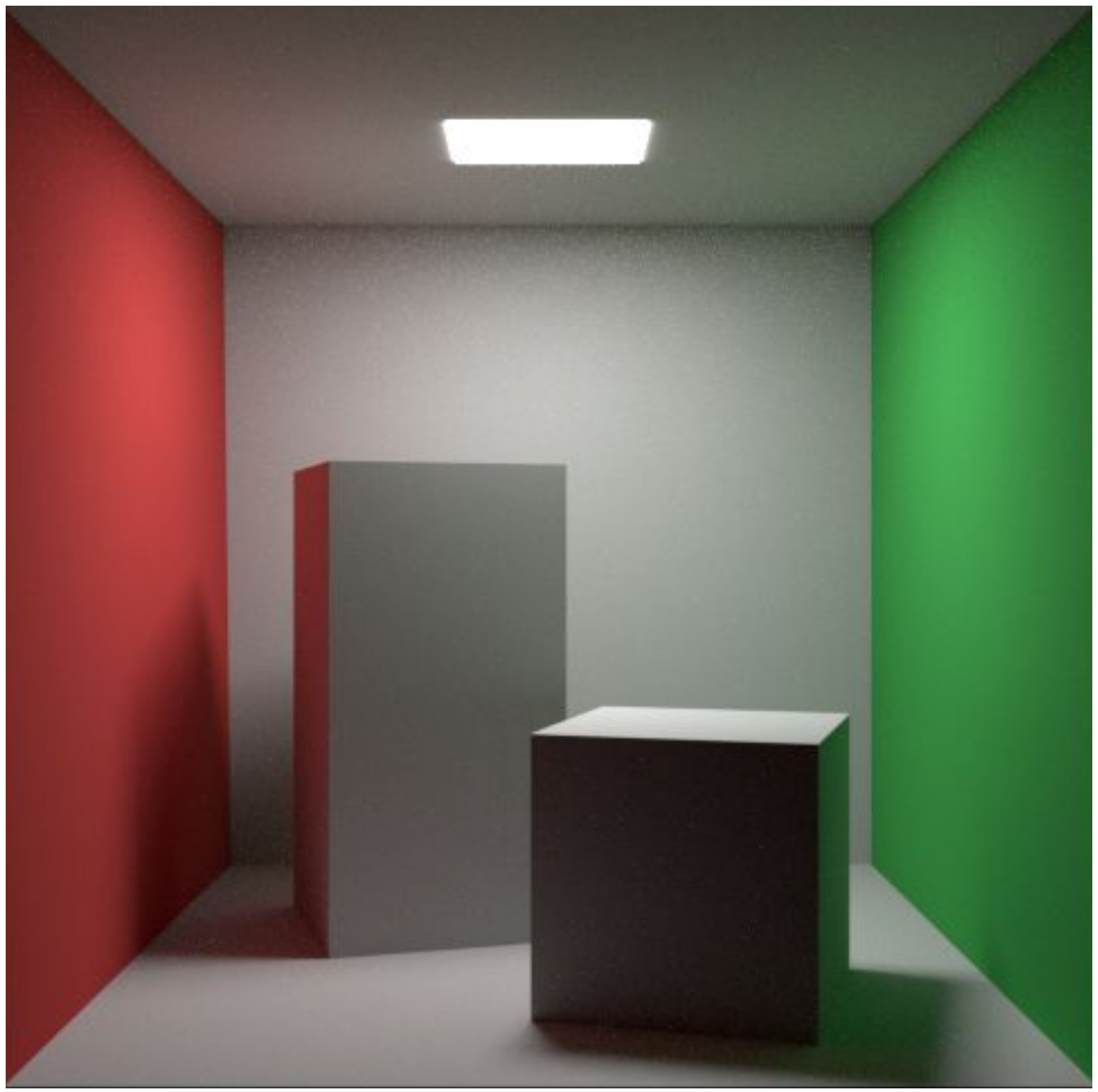


Rendered with 16 spp

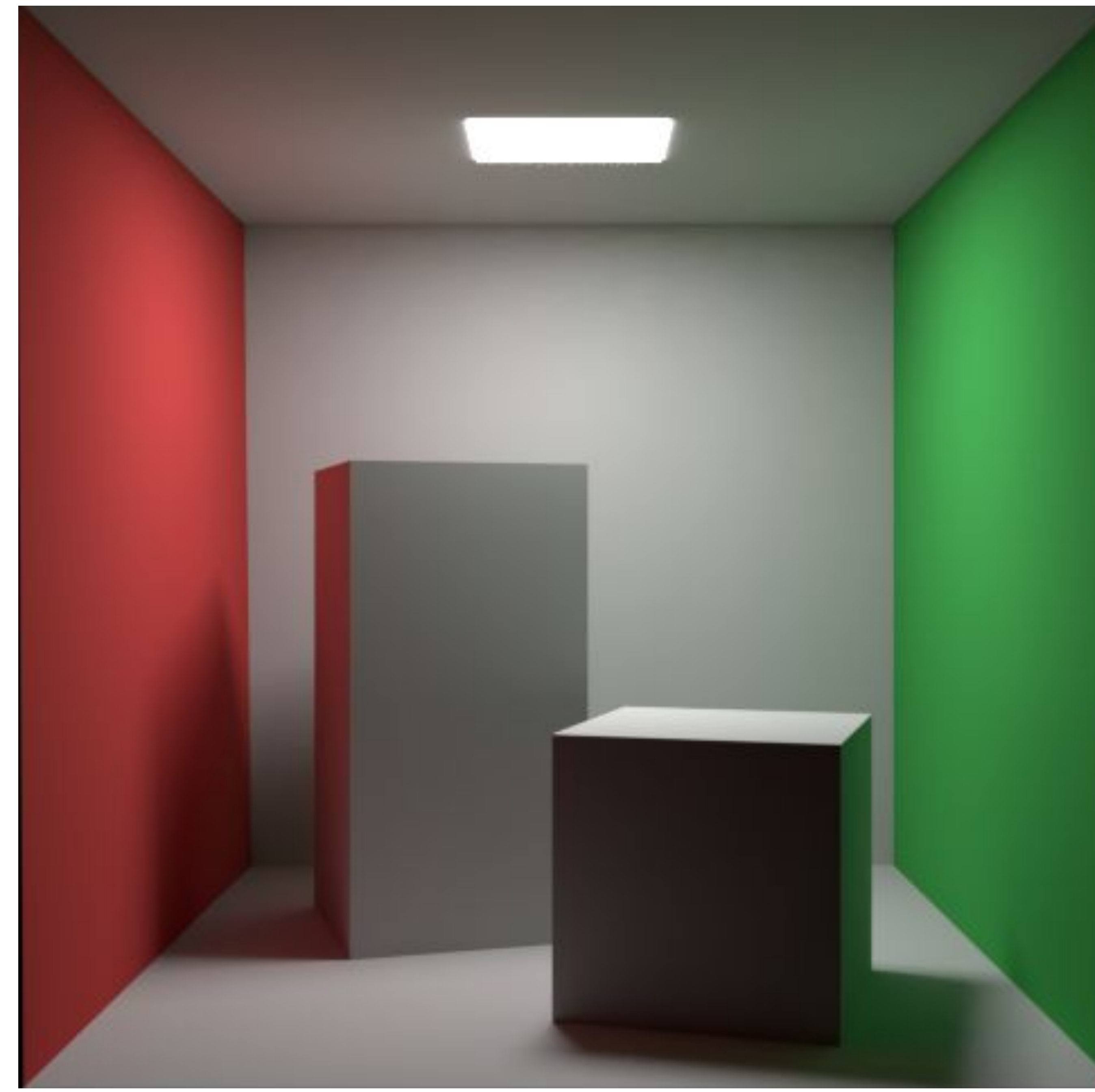
https://openimagedenoise.github.io/gallery.html

Noise Reduction for Cornell Box

This cornell box is created using Blender, and denoised by Blender's built-in denoiser



512 spp, **without** denoiser



512 spp, **with** denoiser

Tutorial 7: Illumination

- Whitted-style Ray Tracing
- Monte Carlo Ray Tracing
  - Monte Carlo Integration
  - Path Tracing
  - Light Sampling
  - Direct & Indirect Illumination
- Epilogue

Topics not covered in detail (or where to go from here)

- Acceleration structure: find the right object
  - partially covered, see BVH in the Rasterization; volume rendering, see lecture slides
- Ray casting: find the right position
  - partially covered, see Liang's algorithm in the Rasterization, or read code in cornell box task carefully
- Importance sampling
  - advanced(?) math techniques for solving Monte Carlo integration more efficiently
- Noise reduction
  - advanced(?) math techniques for solving Monte Carlo integration with fewer spp ⇒ Real-time ray tracing!
- Global illumination approximation
  - calculate the rendering equation for different geometric structures (e.g. voxel) ⇒ Real-time ray tracing!
- Light transportation variances
  - More physical concern, e.g. photon mapping, metropolis light transport ⇒ Precise accurate offline rendering

Rasterization v.s. Ray Tracing in Real-Time

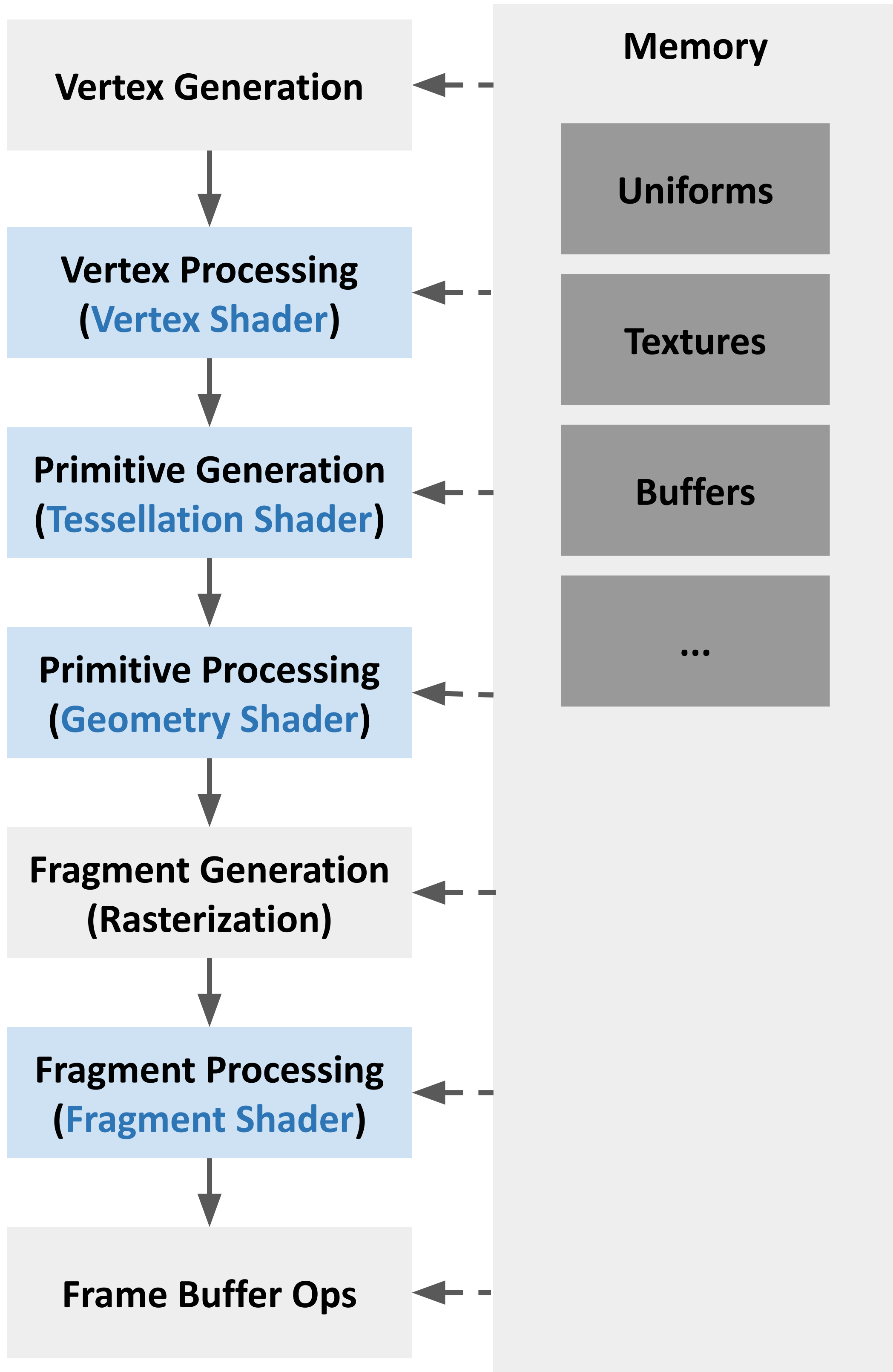
Shaders in the pipeline:

- **Vertex shader**: transforming vertices
- **Tessellation shader**: subdividing meshes
- **Geometry shader**: generating new primitives
- **Fragment shader**: colorizing pixels

Shader not in the pipeline:

- **Compute shader**: for general purpose computing

```
init frame buffer
init z buffer
for each triangle t in scene {
   tp = project(t)
   for each pixel p in frame buffer {
       if tp covers p {
           if z value at p is closer than z buffer at p {
               update z buffer and frame buffer
           }
       }
   }
}
flush frame buffer to monitor
```



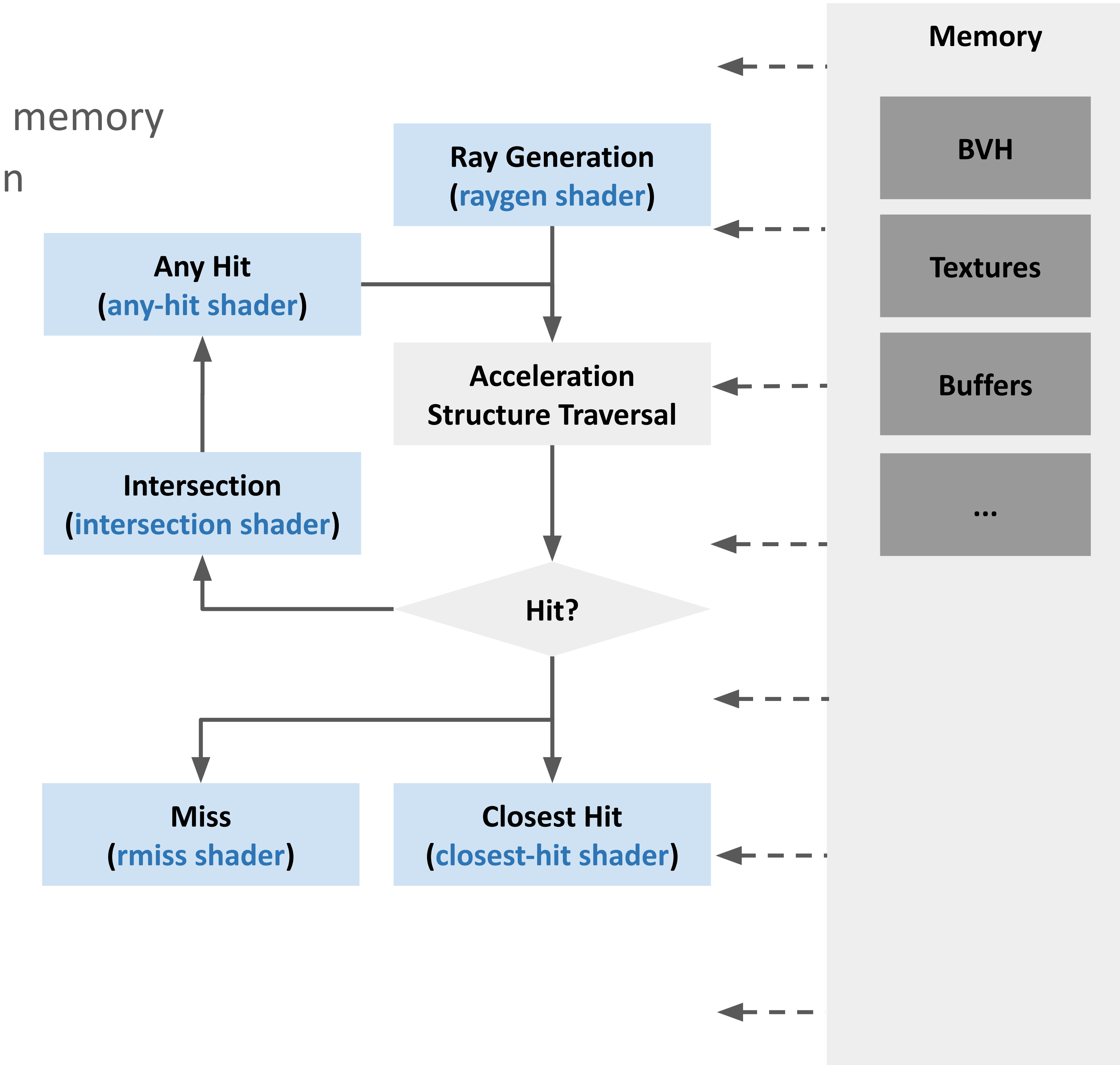
*This pipeline is standardized in OpenGL, NVIDIA's hardware supports an implementation

Rasterization v.s. Ray Tracing in Real-Time

Shaders in the pipeline:

- **Raygen shader**: deal with rays and write final output to memory
- **Intersection shader**: handling ray-primitives intersection
- **Closest-hitit shader**: only on the closest hit position
- **Any-hit shader**: for all possible intersections
- **Rmiss shader**: invoke when no intersection is found

```
init frame buffer
for each pixel p in frame buffer {
   construct a ray from p
   for ray bounces is not over {
       for each triangle t in the scene {
           if ray hit t at x {
               keep x if closest and update the ray
               break
           }
       }
   }
   update frame buffer
}
flush frame buffer to monitor
```



*This pipeline is proposed by NVIDIA, implemented in RTX-series hardwares

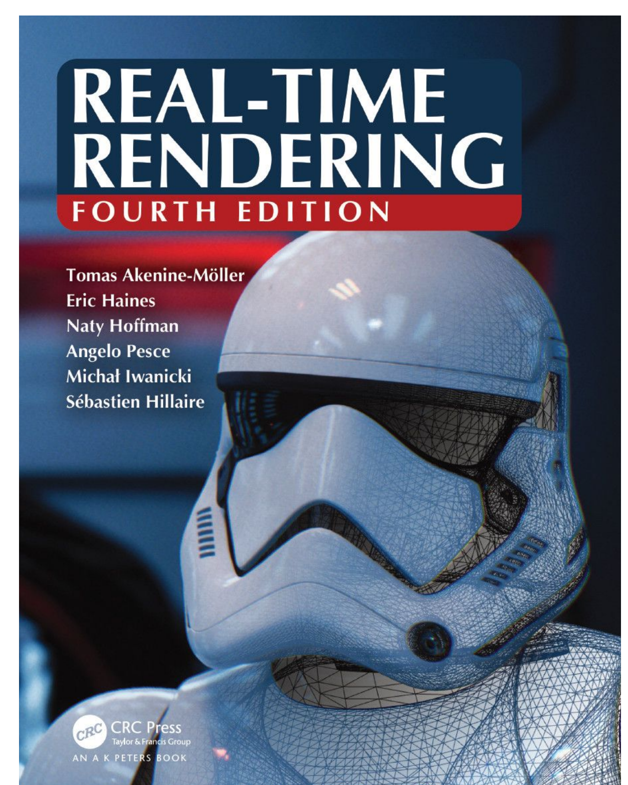
Real-Time Ray Tracing (RTRT) Today

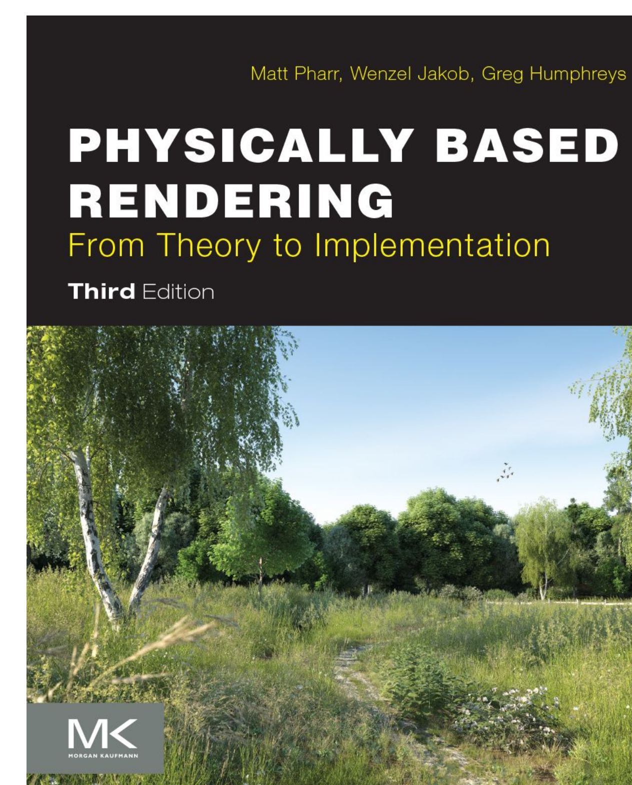
- Real-time rendering related research advances
  - Parallelized BVH construction and traversal algorithm research (approx. 2010-2013)
  - Deep neural networks based image denoising research (approx. 2008-2017)
  - Voxel-based global illumination research (approx. 2005-2012)
  - …
- Industrial practices
  - NVIDIA's RTX hardware implementation (2018-today)
    - Works for PCs
  - Epic Games' UE5 software implementation (2020-today)
    - Works for PlayStation 5, XBOX, ...

https://twitter.com/paniq/status/977582718040014848/photo/1

Take Away

- Path tracing is old fashioned (1986) but still the mostly used global illumination solution for industrial photorealistic rendering, and have been largely applied for decades!
- Too many exciting advances which the course is too basic/short to contain :)
- RTRT is replacing (?) rasterization over the next generation (decade)
- We don't know the future, but what do you think?
- Check these books if you still interested in CG and want dive further:





Thanks!
What are your questions?