

Praktikum

Geometry Processing

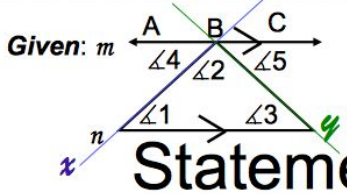
1 Introduction

Ludwig-Maximilians-Universität München

Session 1: Introduction

- Motivation
- Recap: Graphics Pipelines
- Geometry Representations
- Blender Basics
- Summary

This course is *not* about ...

 <p>Given: m</p> <p>Statement</p>	<p>Proof of Triangle Angle Sum Theorem</p> <p>Given: m & n parallel.</p> <p>Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$</p> <p>Reason</p>
<ol style="list-style-type: none"> 1) Lines m and n are <i>parallel</i> 2) $\angle ABC$ is a <i>Straight</i> angle. 3) $m\angle ABC = 180^\circ$ 4) $m\angle 4 + m\angle 2 + m\angle 5 = m\angle ABC$ 5) $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$ 6) x is <i>transversal</i> forming $\angle 1$ & $\angle 4$ y is <i>transversal</i> forming $\angle 3$ & $\angle 5$ 7) $\angle 1$ & $\angle 4$ are <i>alternate</i> Int. \angles 8) $\angle 3$ & $\angle 5$ are Alternate Int. \angles 9) $\angle 1 \cong \angle 4$ & $\angle 3 \cong \angle 5$ 10) $m\angle 1 = m\angle 4$ & $m\angle 3 = m\angle 5$ 11) $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ 	<ol style="list-style-type: none"> 1) <i>Given</i> 2) <i>Definition</i> of Straight Angle 3) If Straight Angle, then 180 4) Angle Addition Postulate 5) Substitution <i>Property</i> of <i>Equality</i> 6) Definition of Transversal(s) 7) Definition of Alt Interior Angles. 8) Definition of Alt Interior Angles 9) If <i>parallel</i> transversal then <i>congruent</i> Alt. Int. \angle 10) Definition of <i>congruent</i> Angles 11) Substitution Property of = <p style="text-align: center;">QED</p>

This course is *not* about ...

Given: m

Statement

Proof of Triangle Angle Sum Theorem

Given: m & n parallel.

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

Reason

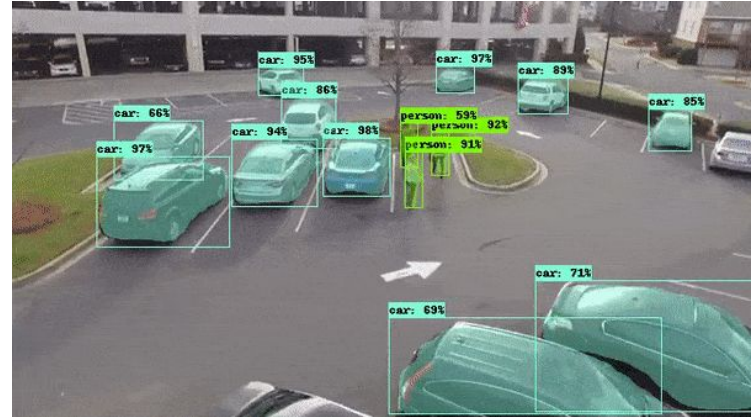
Statement	Reason
1) Lines m and n are <i>parallel</i> .	1) <i>Given</i>
2) $\angle ABC$ is a <i>Straight</i> angle.	2) <i>Definition</i> of Straight Angle
3) $m\angle ABC = 180^\circ$	3) If Straight Angle, then 180
4) $m\angle 4 + m\angle 2 + m\angle 5 = m\angle ABC$	4) Angle Addition Postulate
5) $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$	5) Substitution <i>Property</i> of <i>Equality</i>
6) x is <i>transversal</i> forming $\angle 1$ & $\angle 4$ y is <i>transversal</i> forming $\angle 3$ & $\angle 5$	6) Definition of Transversal(s)
7) $\angle 1$ & $\angle 4$ are <i>alternate</i> Int. \angle s	7) Definition of Alt Interior Angles.
8) $\angle 3$ & $\angle 5$ are Alternate Int. \angle s	8) Definition of Alt Interior Angles
9) $\angle 1 \cong \angle 4$ & $\angle 3 \cong \angle 5$	9) If ^{parallel} transversal then <i>congruent</i> Alt. Int. \angle
10) $m\angle 1 = m\angle 4$ & $m\angle 3 = m\angle 5$	10) Definition of <i>congruent</i> Angles
11) $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	11) Substitution Property of =

QED

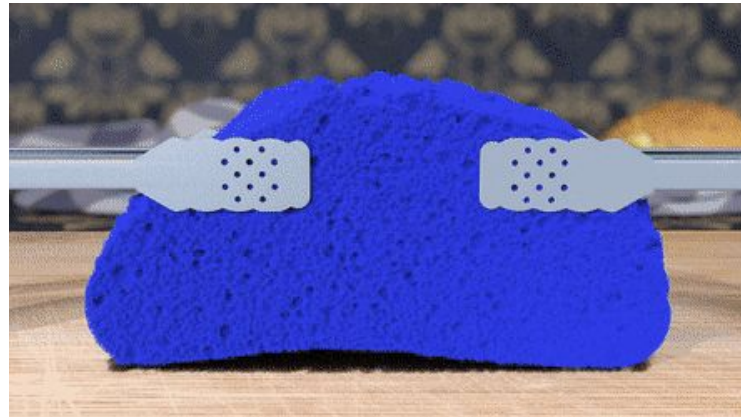
This course is *not* about ...



[Yan et al. 2015]

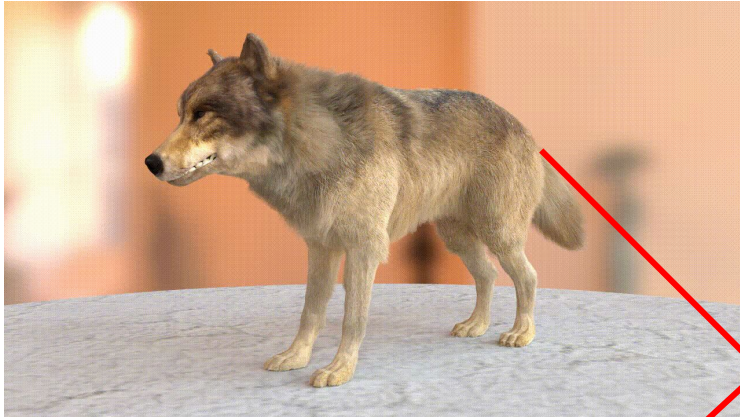


[He et al. 2018]

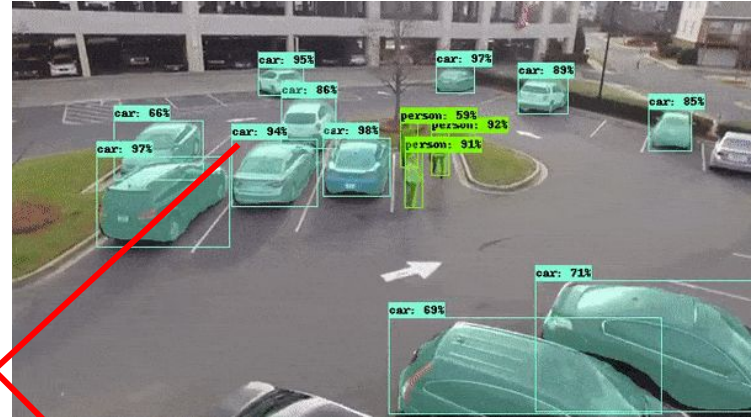


[Wolper et al. 2019]

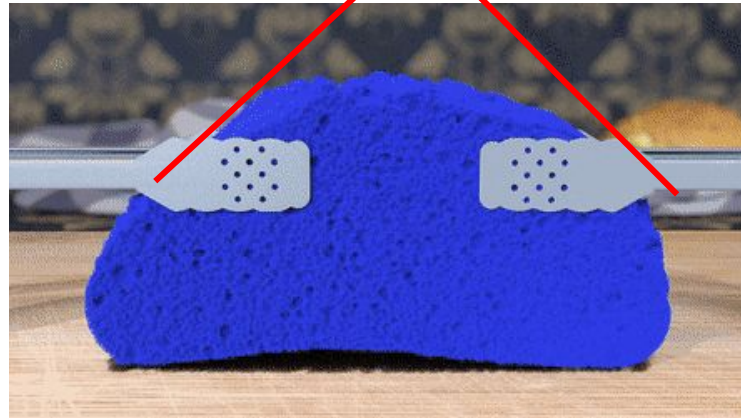
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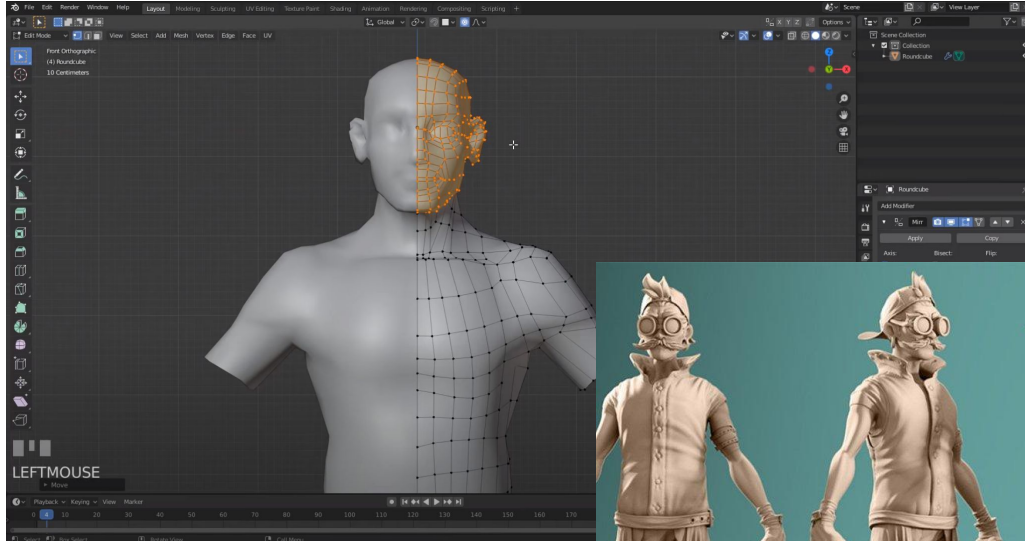


[He et al. 2018]

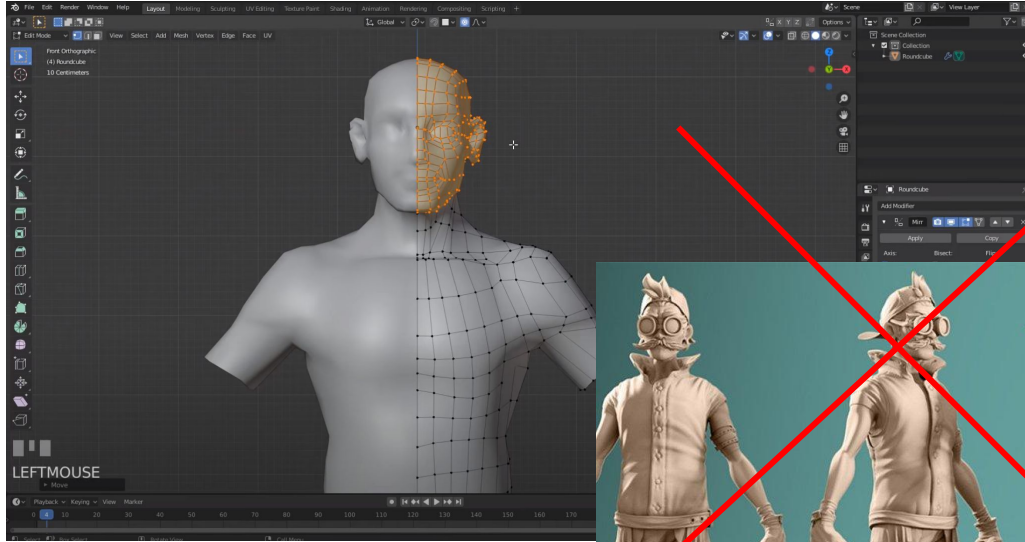


[Wolper et al. 2019]

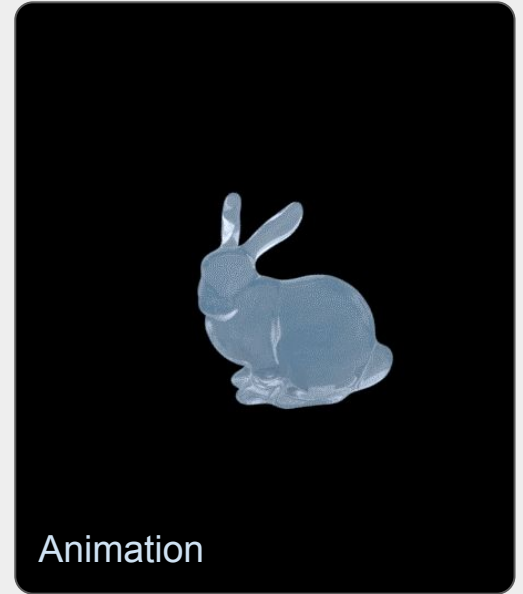
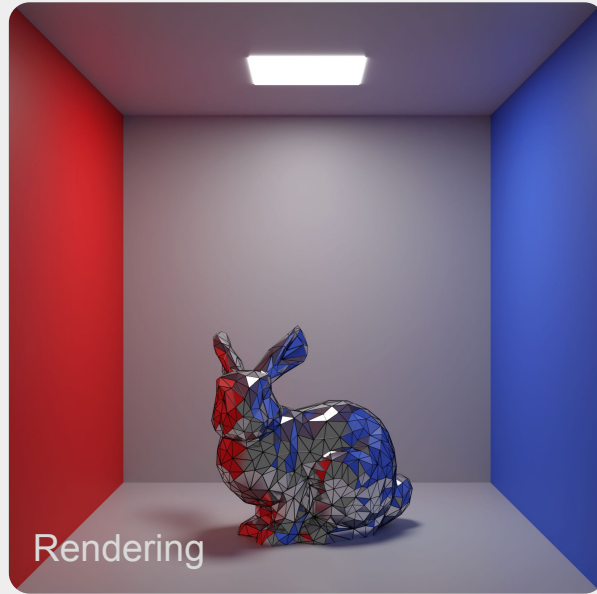
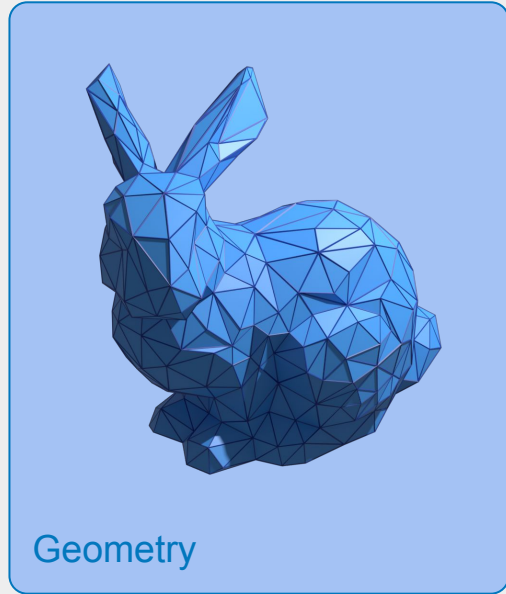
This course is also *not* about ...



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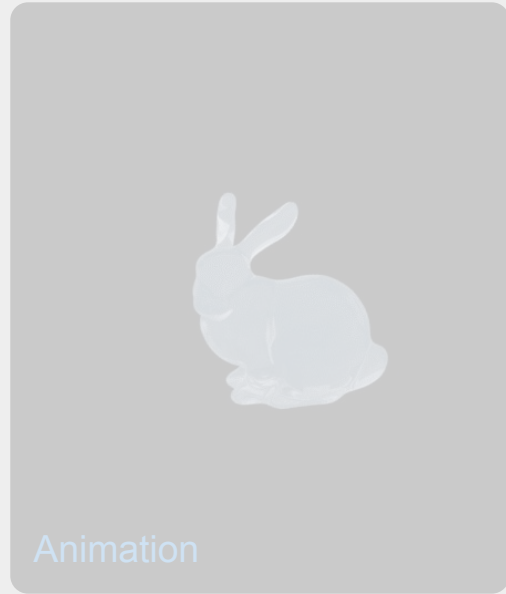
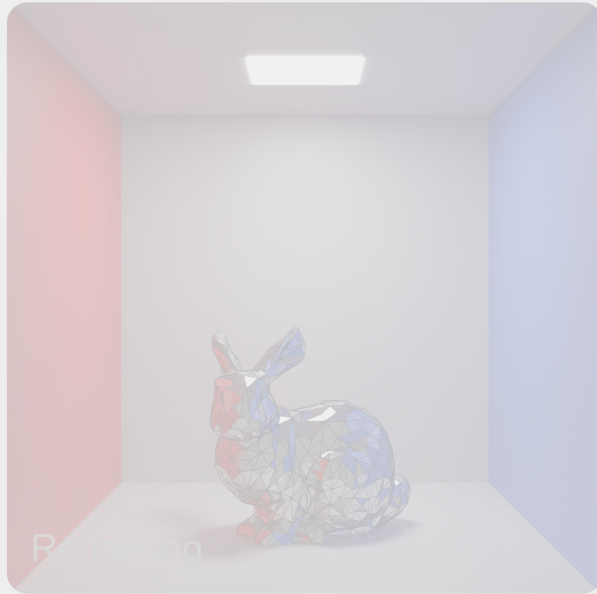
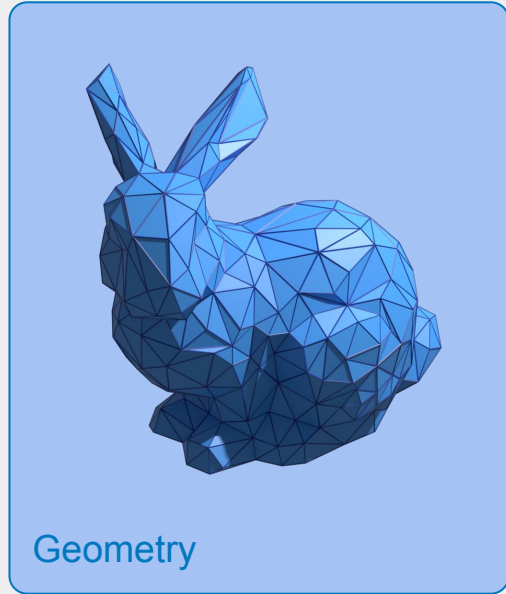


This course is a direct extend to the **CG1** for geometry



Computer Graphics

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Computer Graphics

We will Focus on How to Deal with 3D Geometries *Algorithmically*



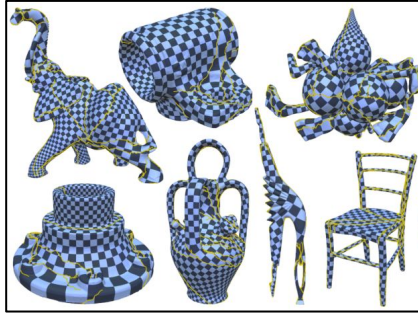
Curvature;

We will Focus on How to Deal with 3D Geometries *Algorithmically*



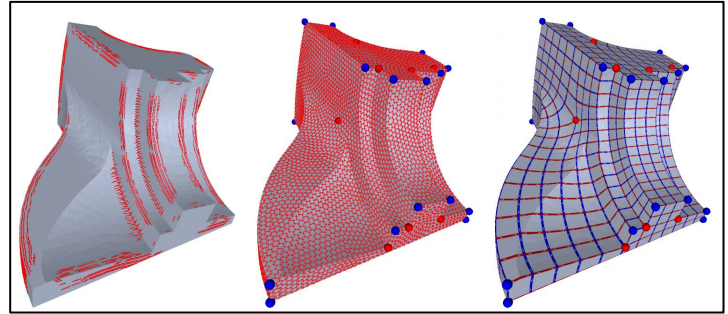
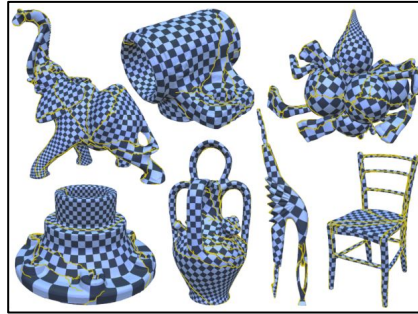
Curvature; Smoothing;

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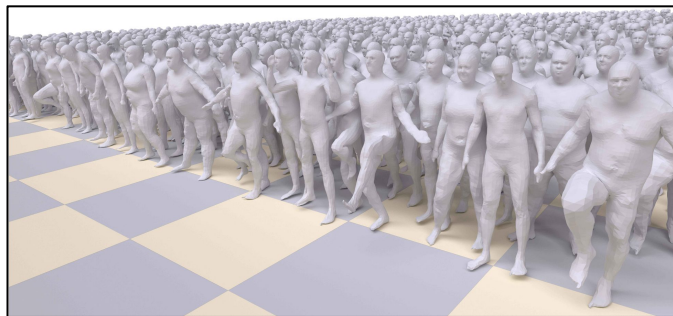
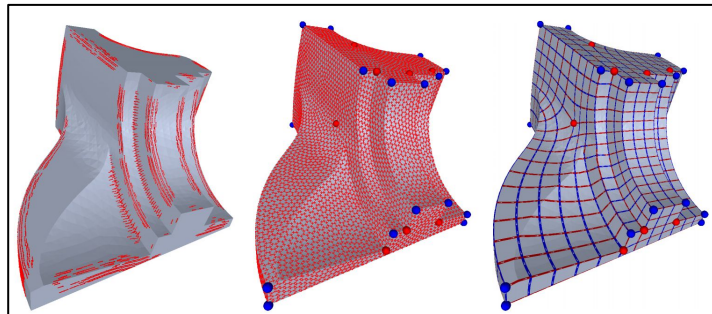
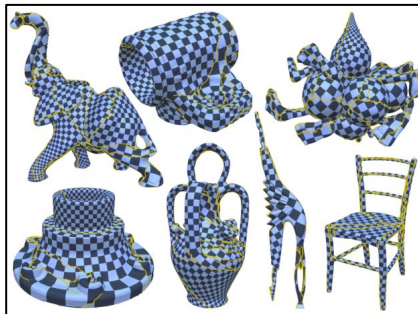
Curvature; Smoothing; Parameterization;

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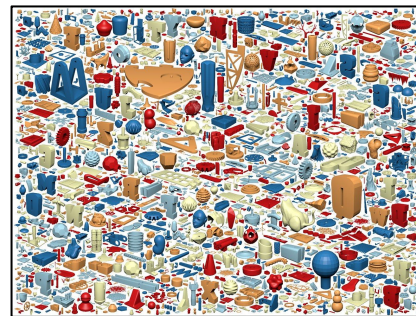
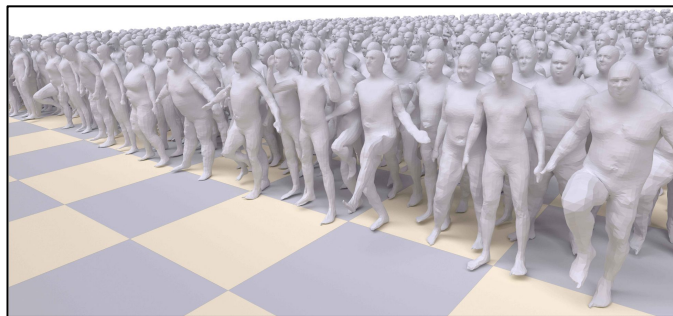
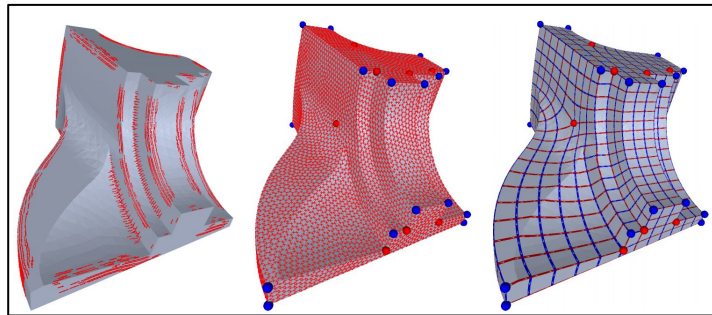
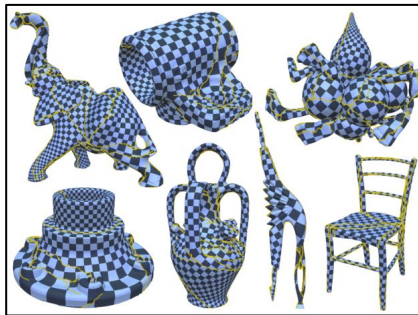
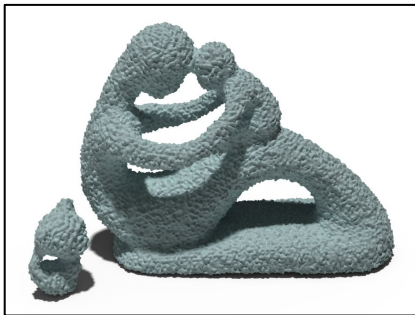
Curvature; Smoothing; Parameterization; Remeshing;

We will Focus on How to Deal with 3D Geometries *Algorithmically*



Curvature; Smoothing; Parameterization; Remeshing; Deformation;

We will Focus on How to Deal with 3D Geometries *Algorithmically*



.....

Curvature; Smoothing; Parameterization; Remeshing; Deformation; Shape Analysis; ...

This Course remains Interdisciplinary ...

$$\mathcal{H}(\mathcal{M}, \mathcal{M}') = \sqrt{\frac{1}{|\mathcal{S}|} \iint_{v \in \mathcal{S}} d(p, \mathcal{S}')^2 d\mathcal{S}}$$

Mathematics

Formal foundation: **Differential geometry**, numeric analysis, ...
(back to 19 century)



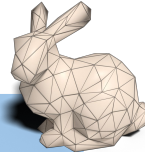
Geometry Processing

This Course remains Interdisciplinary ...

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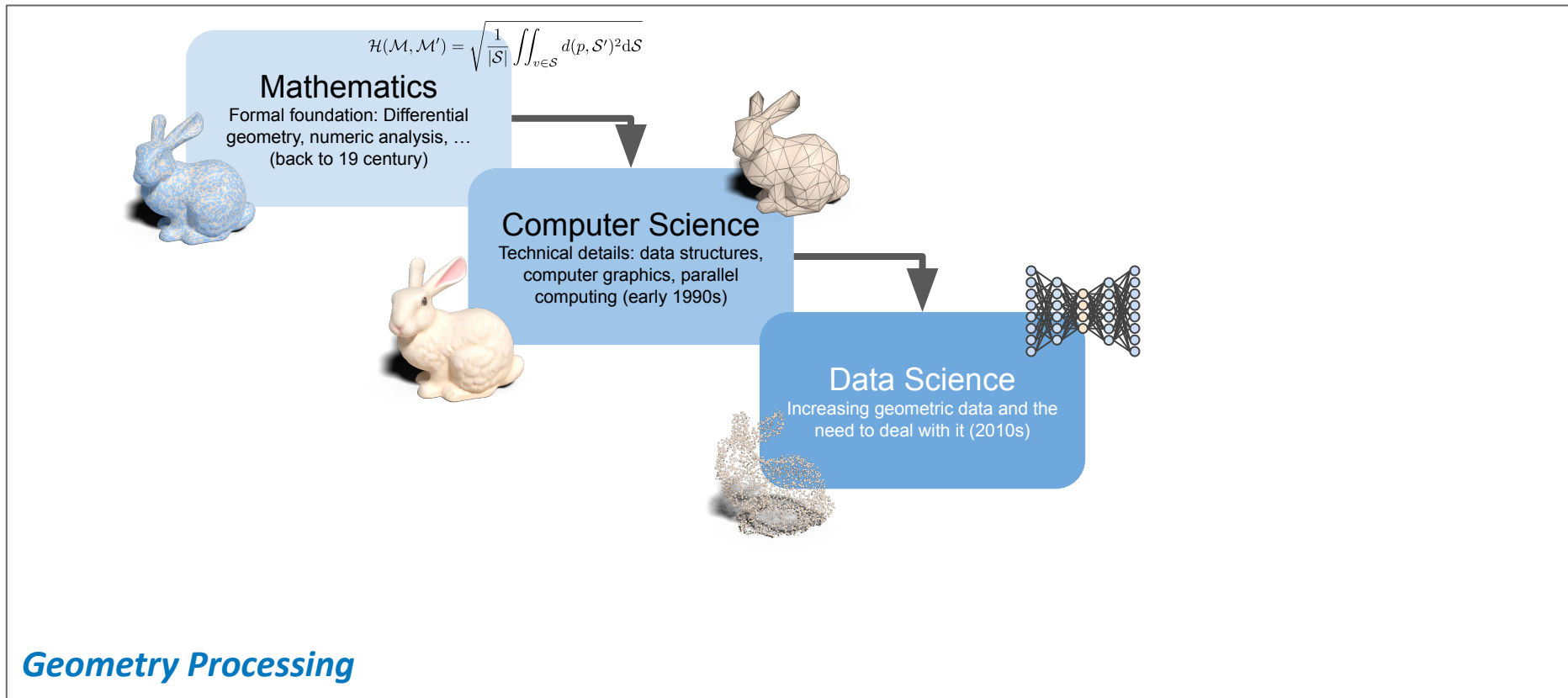
Computer Science

Technical details: data structures, computer graphics, parallel computing (early 1990s)

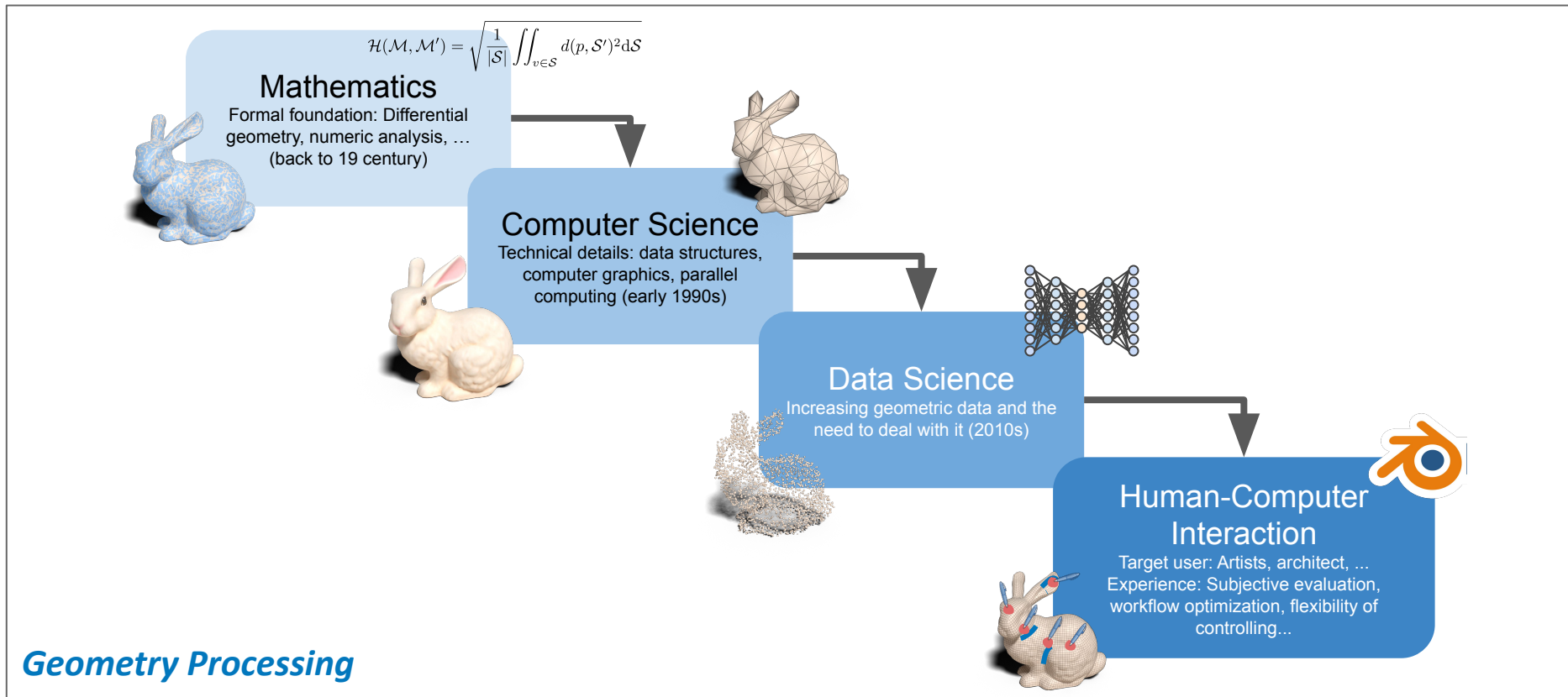


Geometry Processing

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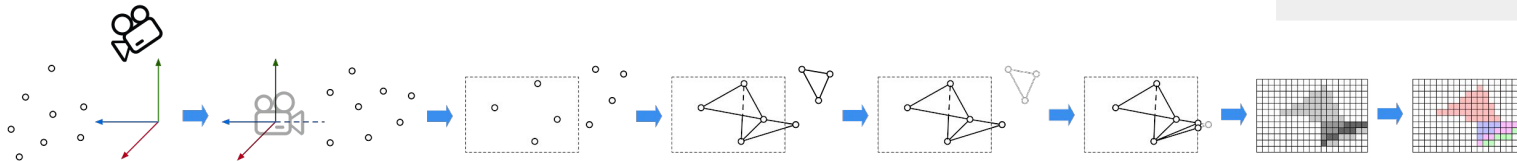
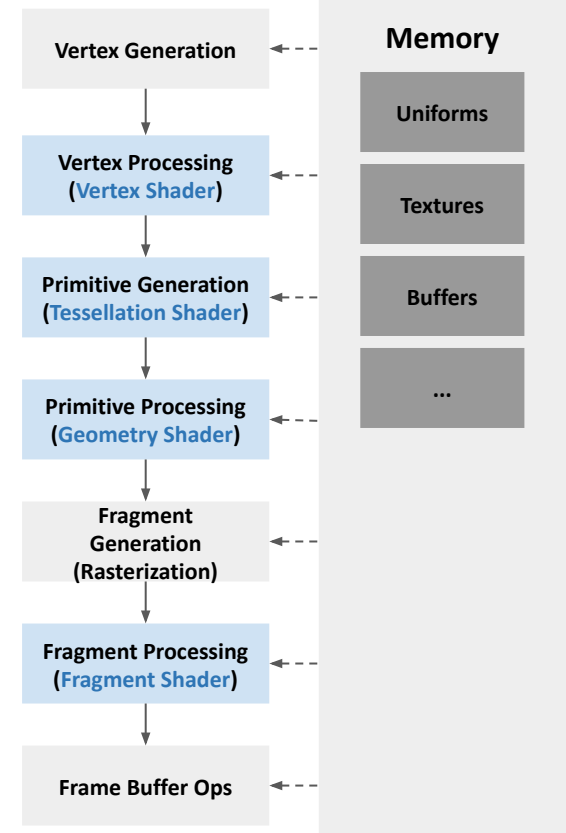


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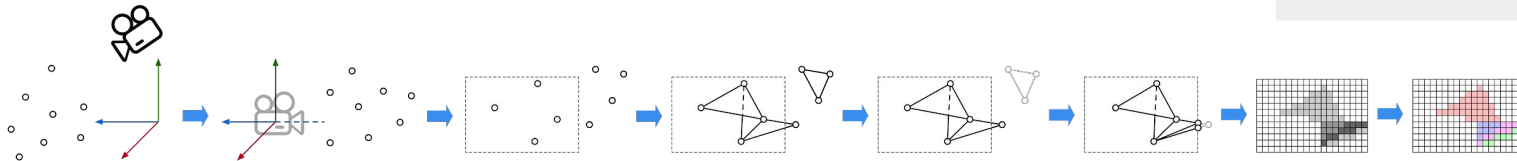
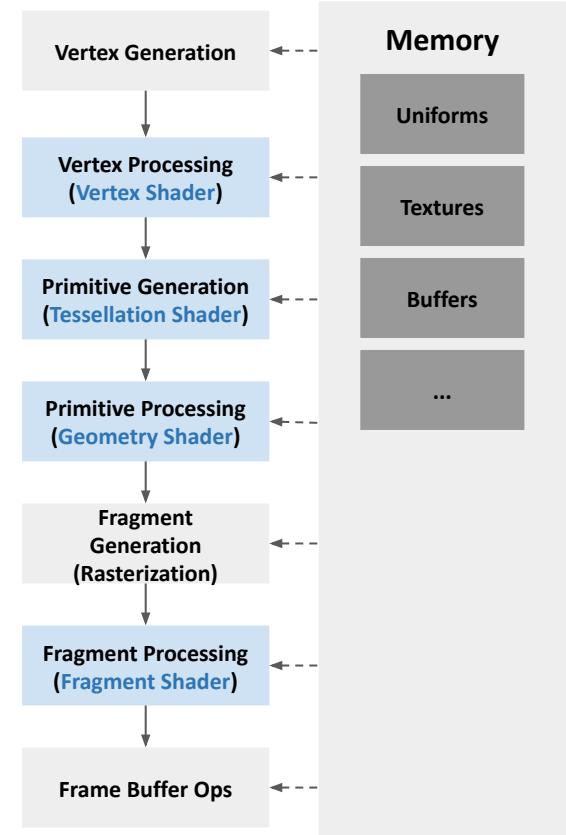
Rasterization Pipeline

```
init frame buffer
init depth buffer
for each triangle t in scene {
    tp = project(t)
    for each pixel p in frame buffer {
        if tp covers p {
            if z passes depth test at p {
                update z buffer and frame buffer
            }
        }
    }
}
flush frame buffer to monitor
```



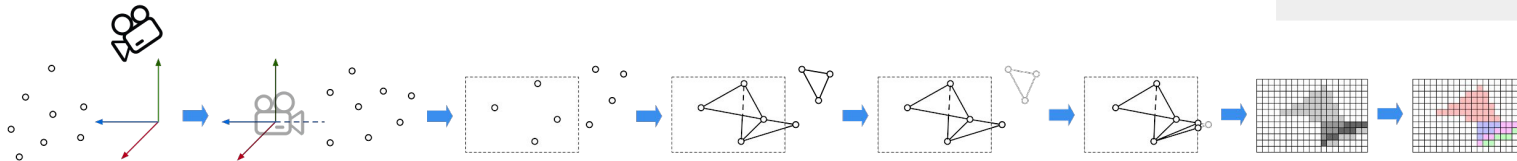
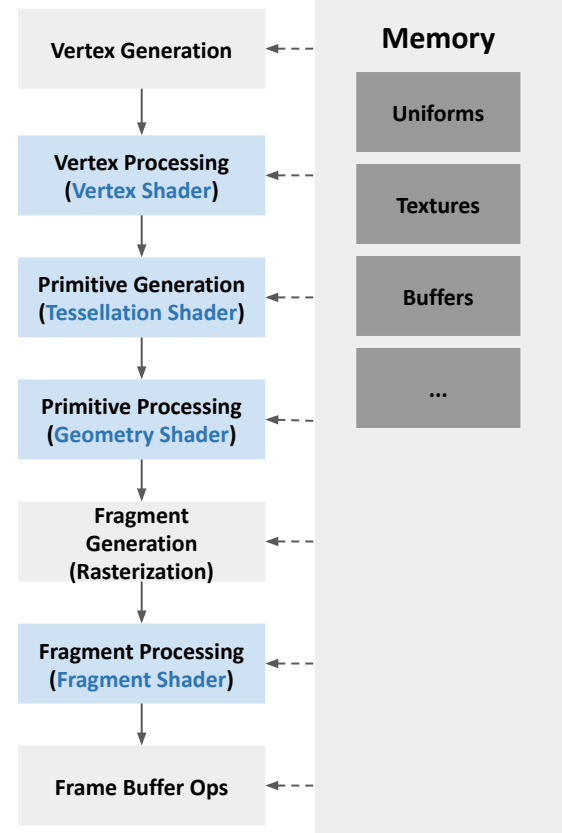
Rasterization Pipeline

```
init frame buffer
init depth buffer
for each triangle t in scene {
    tp = project(t) // MVP
    for each pixel p in frame buffer {
        if tp covers p {
            if z passes depth test at p {
                update z buffer and frame buffer
            }
        }
    }
}
flush frame buffer to monitor
```



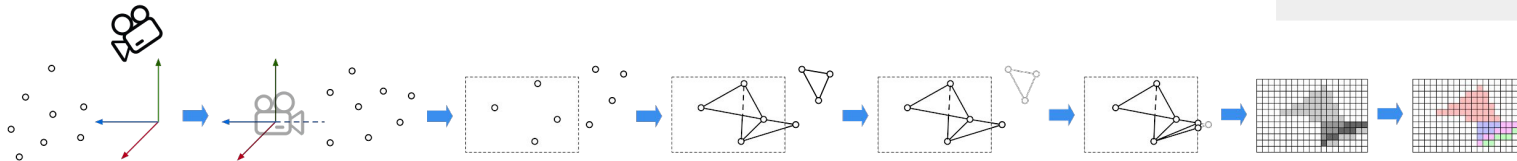
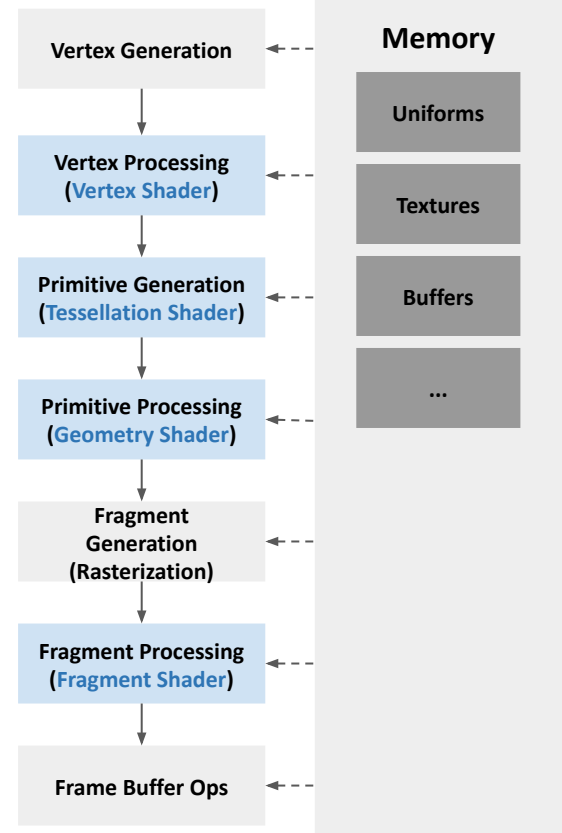
Rasterization Pipeline

```
init frame buffer
init depth buffer
for each triangle t in scene {
    tp = project(t) // MVP
    for each pixel p in frame buffer {
        if tp covers p { // culling
            if z passes depth test at p {
                update z buffer and frame buffer
            }
        }
    }
}
flush frame buffer to monitor
```



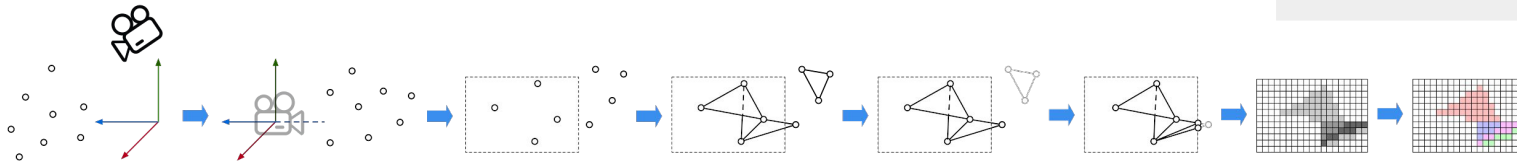
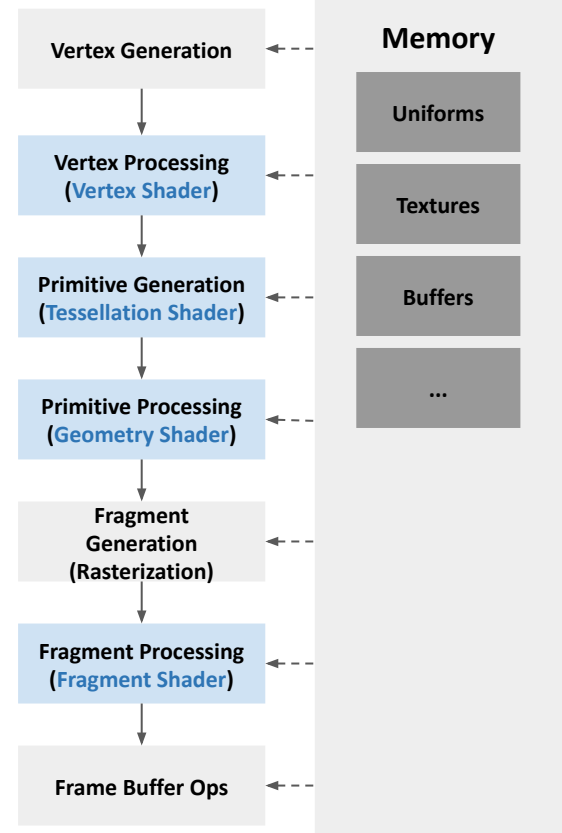
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    for each pixel p in frame buffer {
        if tp covers p { // culling
            if z passes depth test at p { // depth-test
                update z buffer and frame buffer
            }
        }
    }
}
flush frame buffer to monitor
```



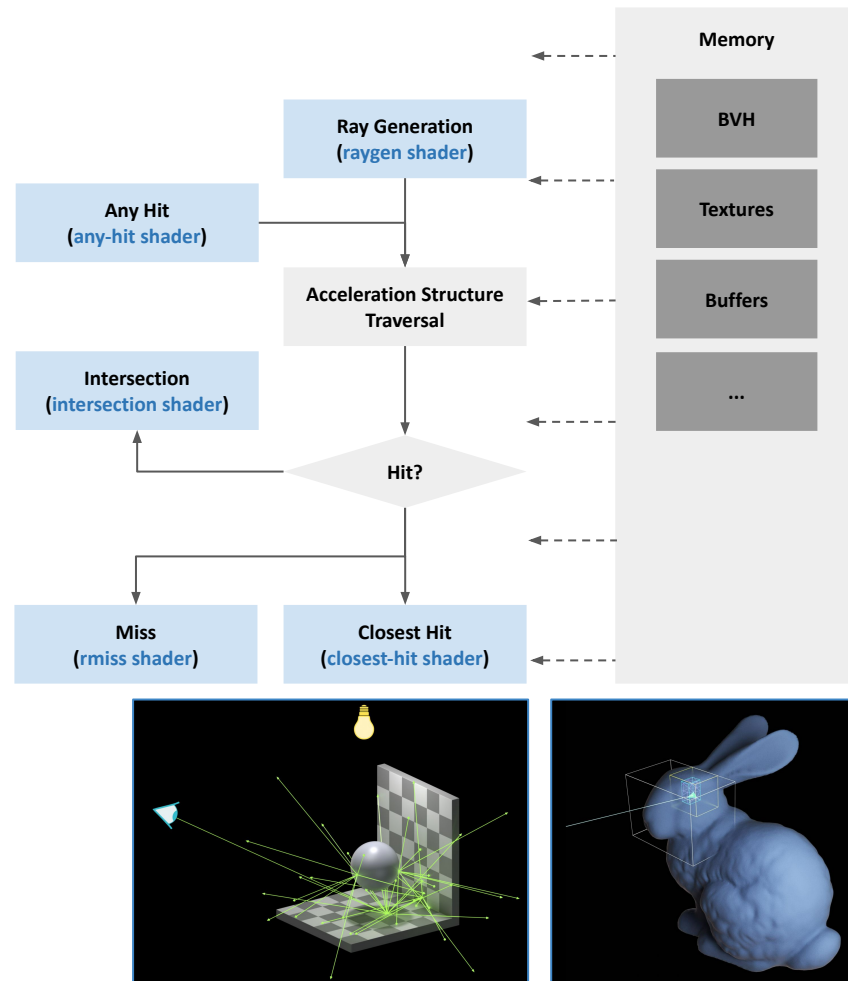
Rasterization Pipeline

```
init frame buffer
init depth buffer
for each triangle t in scene {
    tp = project(t) // MVP
    for each pixel p in frame buffer {
        if tp covers p { // culling
            if z passes depth test at p { // depth-test
                update z buffer and frame buffer // interpolation & update
            }
        }
    }
}
flush frame buffer to monitor
```



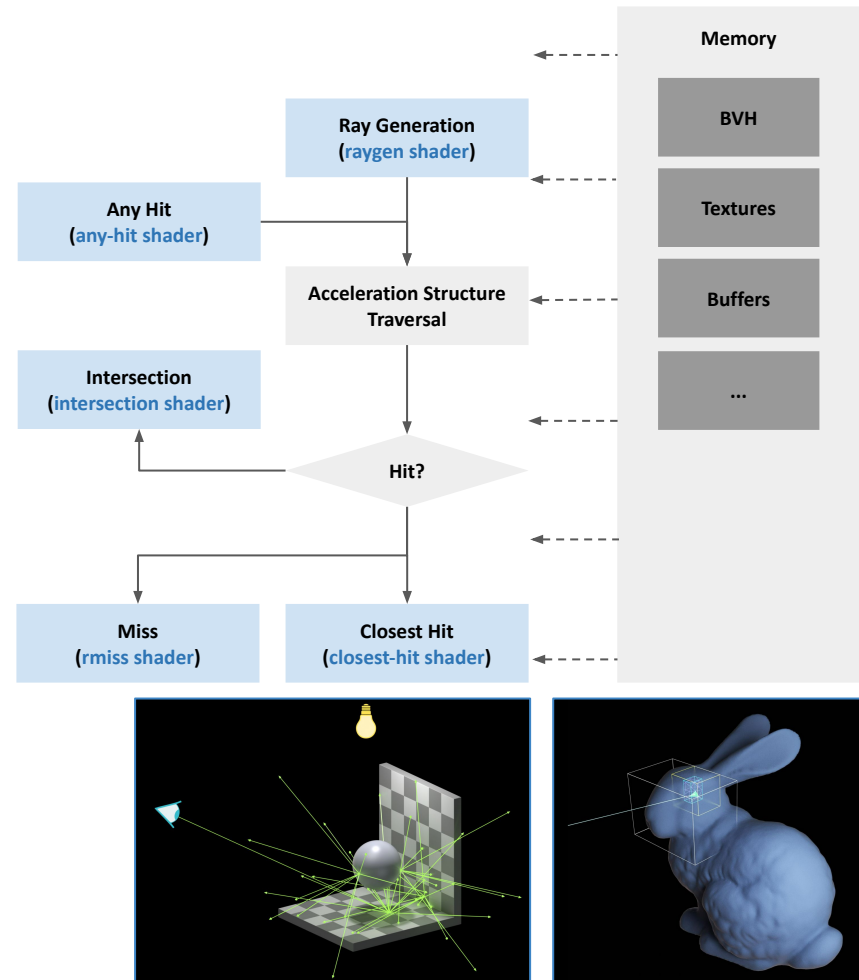
Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
  construct a ray from p
  for ray bounces is not over {
    for each triangle t in the scene {
      if ray hit t at x {
        keep x if closest and update the ray
        break
      }
    }
  }
  update frame buffer
}
flush frame buffer to monitor
```



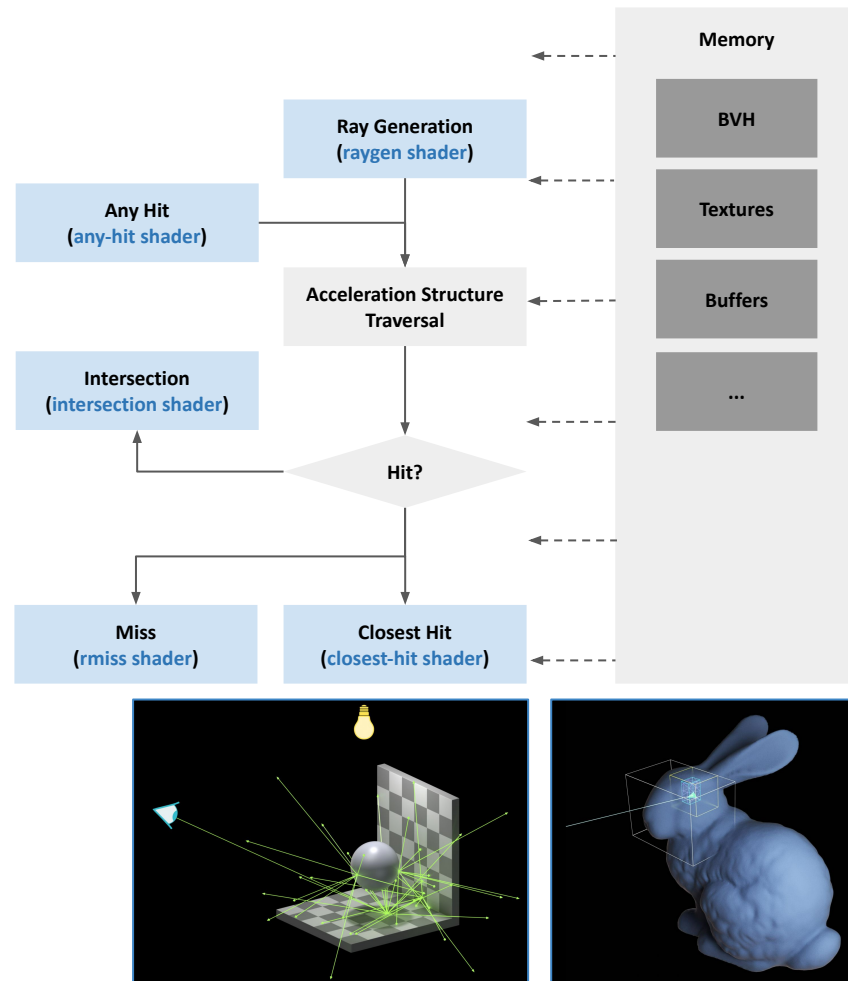
Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
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  for ray bounces is not over {
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        break
      }
    }
  }
  update frame buffer
}
flush frame buffer to monitor
```



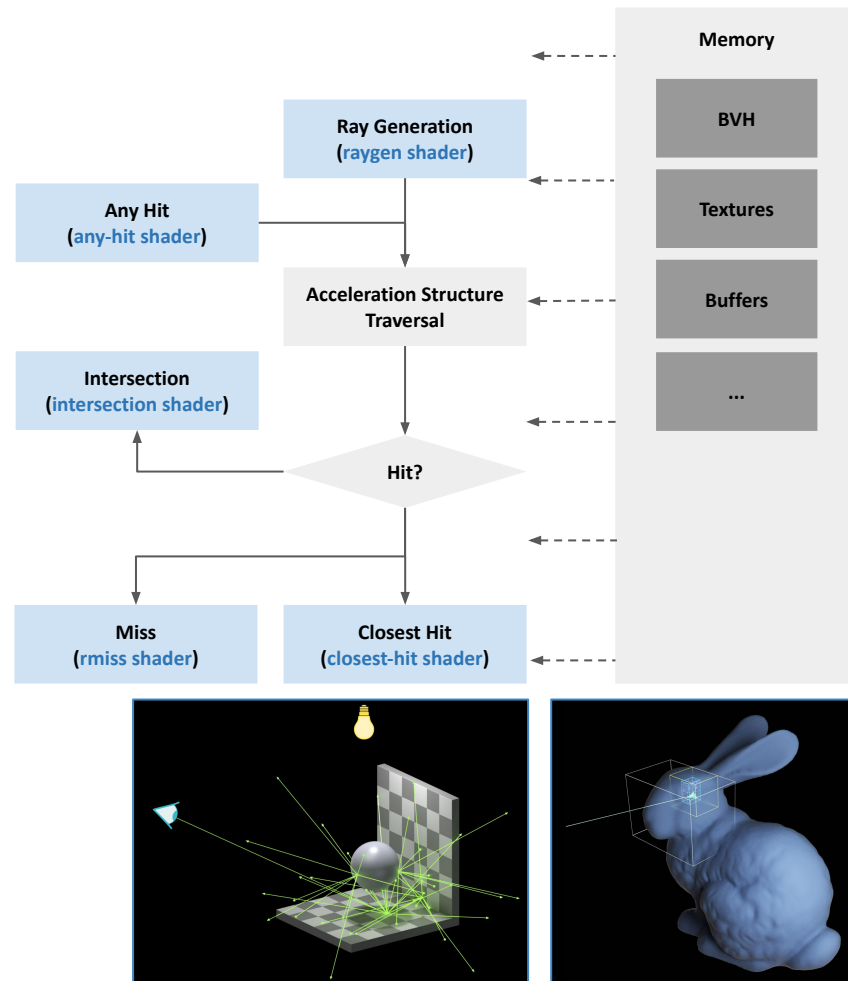
Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
  construct a ray from p // ray generation
  for ray bounces is not over { // russian roulette
    for each triangle t in the scene {
      if ray hit t at x {
        keep x if closest and update the ray
        break
      }
    }
  }
  update frame buffer
}
flush frame buffer to monitor
```



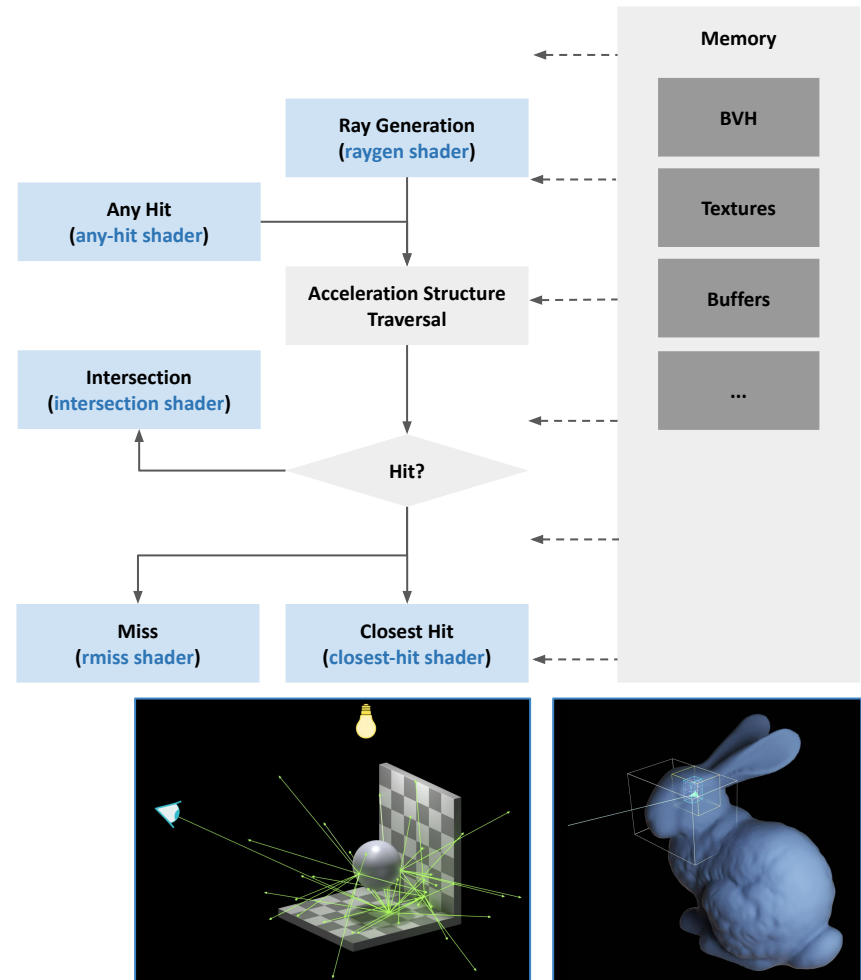
Ray Tracing Pipeline

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init frame buffer
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  construct a ray from p // ray generation
  for ray bounces is not over { // russian roulette
    for each triangle t in the scene { // BVH
      if ray hit t at x {
        keep x if closest and update the ray
        break
      }
    }
  }
  update frame buffer
}
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```



Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
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  for ray bounces is not over { // russian roulette
    for each triangle t in the scene { // BVH
      if ray hit t at x { // ray casting
        keep x if closest and update the ray
        break
      }
    }
  }
  update frame buffer
}
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```



Unanswered Questions (in CG1)

- How geometric objects are created/loaded?

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Let's restart from the very beginning...

Geometry Processing Pipeline



Geometry Processing Pipeline



Scan



Processing

Geometry Processing Pipeline



Scan



Processing

Render



Print



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Representations of Geometry Objects

Point cloud

Voxels

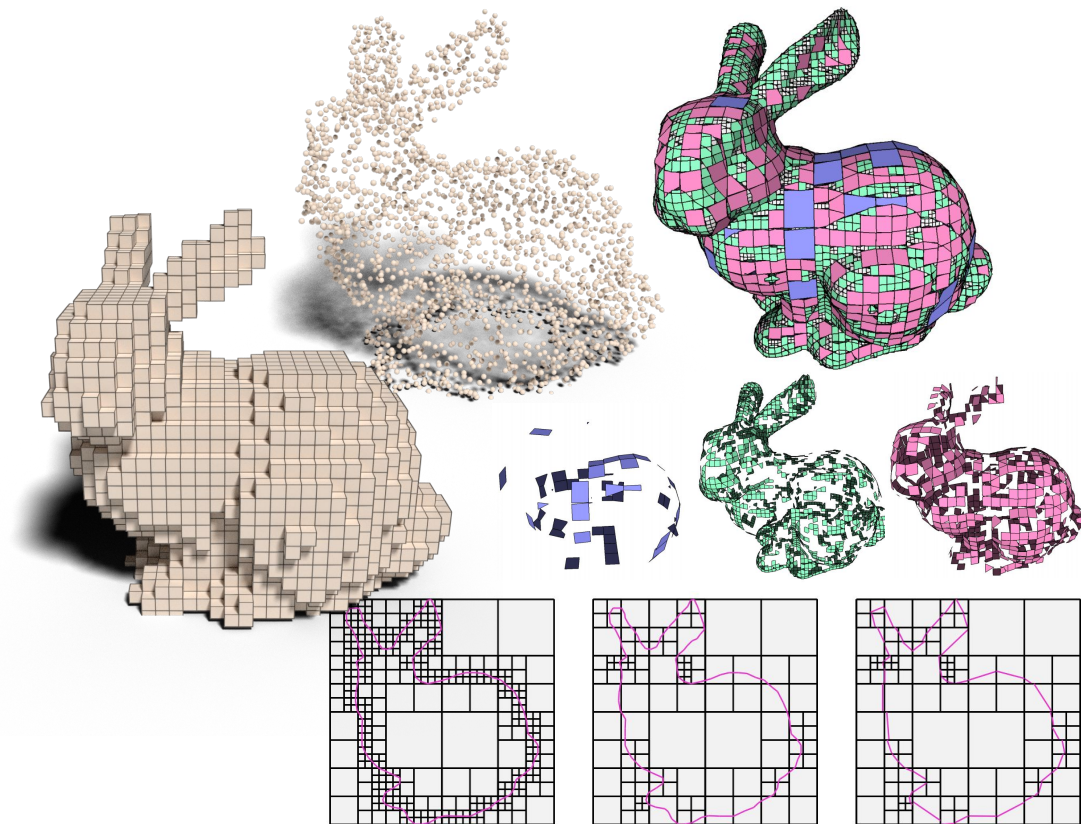
Patches

Implicit

Explicit

Parametric

...



Representations of Geometry Objects

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Patches

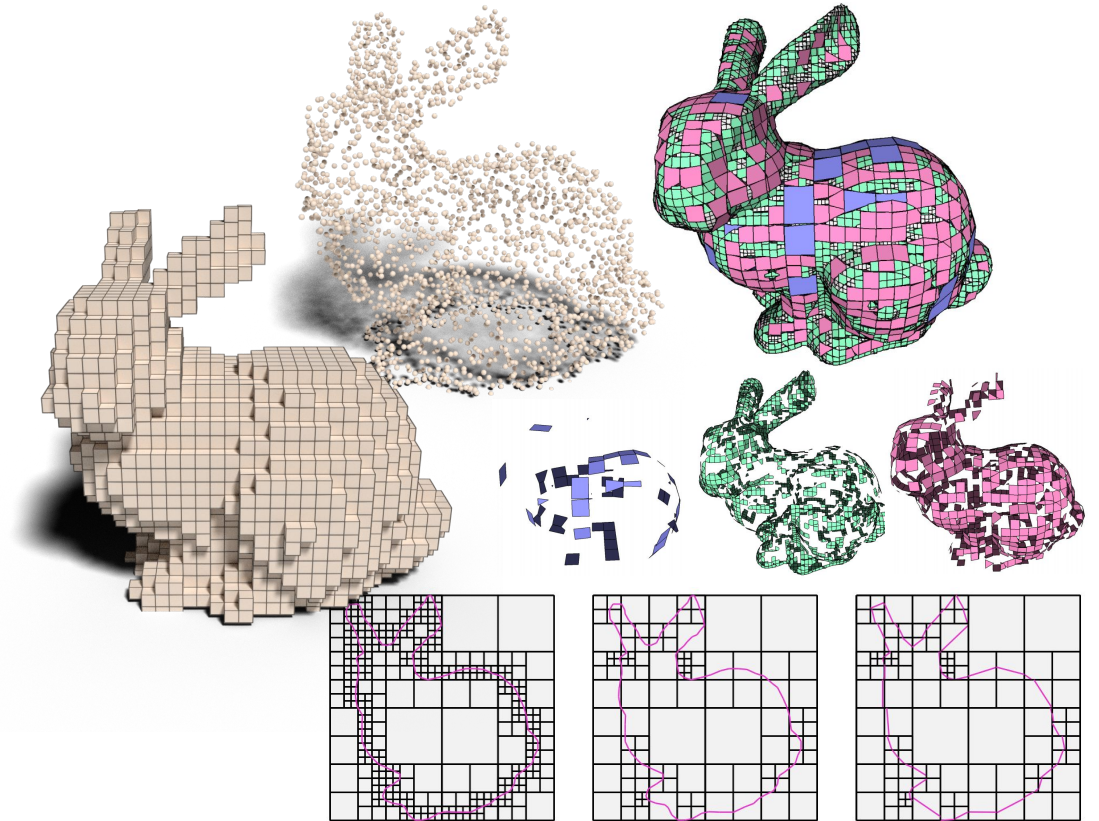
Implicit

Explicit

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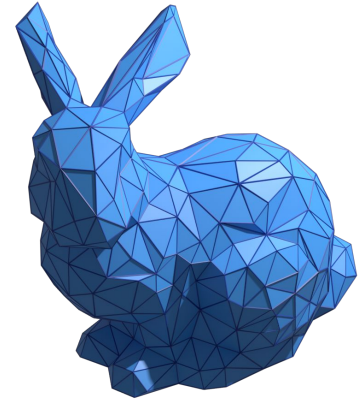
Mesh-based Surface

...



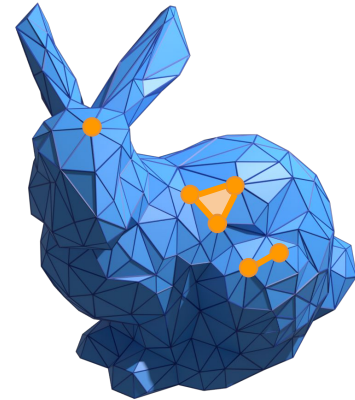
Polygonal Mesh

- A collection of polygons: a segment of a piecewise linear surface representation



Polygonal Mesh

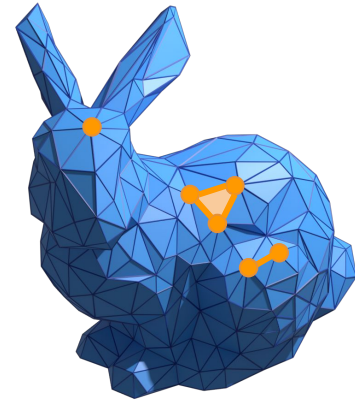
- A collection of polygons: a segment of a piecewise linear surface representation
- *Geometrical* component
 - Vertices $\mathcal{V} = \{v_1, v_2, \dots, v_V\}, v_i \in \mathbb{R}^3$
- *Topological* components
 - Faces $\mathcal{F} = \{f_1, f_2, \dots, f_F\}$
 - Edges $\mathcal{E} = \{e_1, e_2, \dots, e_E\}$
- A polygonal mesh can be formulated as $\mathcal{M} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$



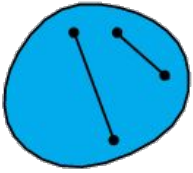
Polygonal Mesh

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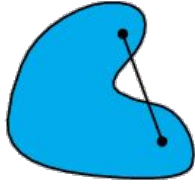
Q: Why are meshes different from graphs?



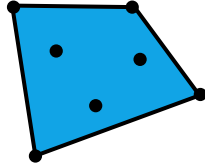
Terminologies



Convex



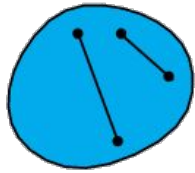
Non-convex



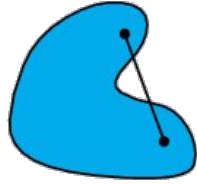
Convex Hull

- *Convex and Convex Hull*

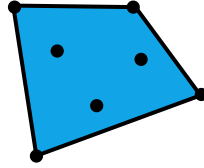
Terminologies



Convex



Non-convex



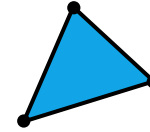
Convex Hull



0-simplex

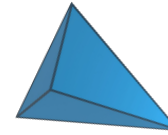


1-simplex



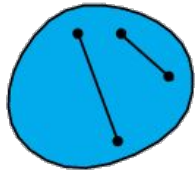
2-simplex

- *Convex and Convex Hull*
- *k-Simplex*: the convex hull of $k+1$ affine-independent vertices
 - e.g. tetrahedra is a **3-simplex**

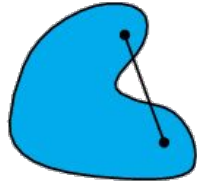


3-simplex

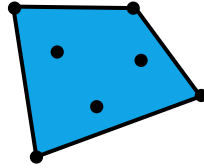
Terminologies



Convex



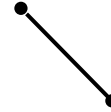
Non-convex



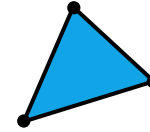
Convex Hull



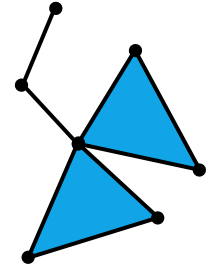
0-simplex



1-simplex

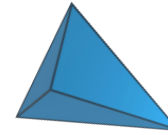


2-simplex



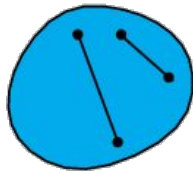
simplicial complex

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- *Face*: any simplices of a subset of vertices

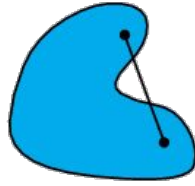


3-simplex

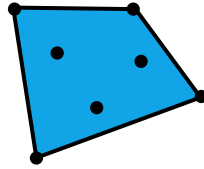
Terminologies



Convex



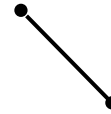
Non-convex



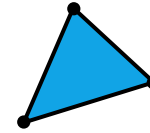
Convex Hull



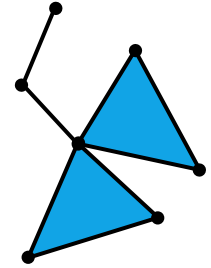
0-simplex



1-simplex

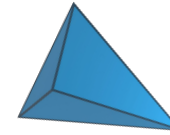


2-simplex



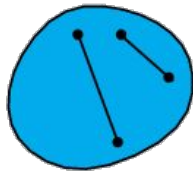
simplicial complex

- *Convex* and *Convex Hull*
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 - e.g. Graph is simplicial **1-complexes**, triangle meshes are simplicial **2-complexes**

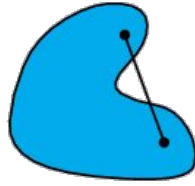


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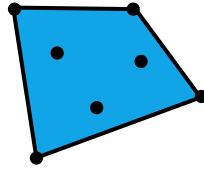
Terminologies



Convex



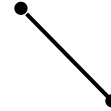
Non-convex



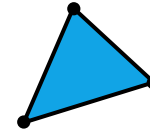
Convex Hull



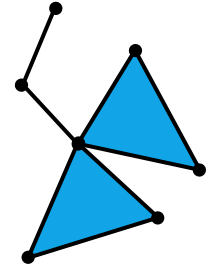
0-simplex



1-simplex

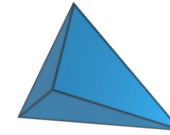


2-simplex

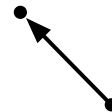


simplicial complex

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- Simplex can have **orientation**

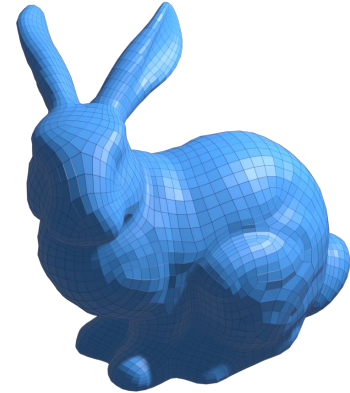
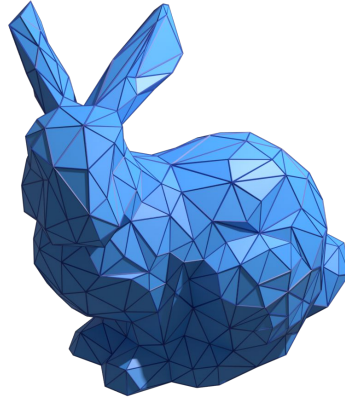


3-simplex



Types of Polygon Meshes

- Triangle Meshes
- Quadrilateral meshes
- ...



Q: What do they have in common?

Typological Invariant: *Euler-Poincaré Formula*

$$F - E + V = 2(1 - g)$$

genus: #holes

Euler characteristic

Euler characteristic allows to check topological property at constant time

e.g. the euler characteristic of a convex polyhedron is 2

Corollary (why?):

- Triangle Mesh
 - $F \approx 2V, E \approx 3V$
 - avg. vertex degree = 6
- Quad Mesh
 - $F \approx V, E \approx 2V$
 - avg. vertex degree = 4



$g=0$



$g=1$



$g=2$



$g=3$

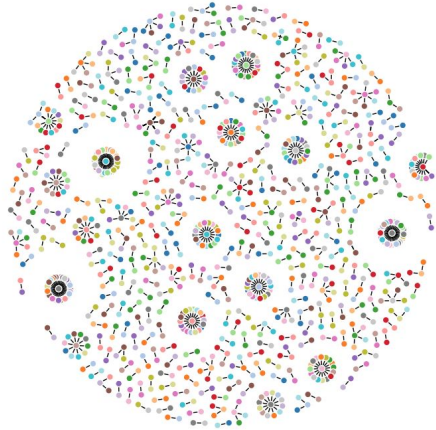
Why Polygons?

- Polygon mesh is a good surface compromise of "physical" solid
 - Think about approximation error (recall Taylor formula from calculus)
- Arbitrary topology
- Flexibility for piecewise smooth surfaces
- Flexibility for adaptive refinement (subdivision)
- Render efficiency
- ...

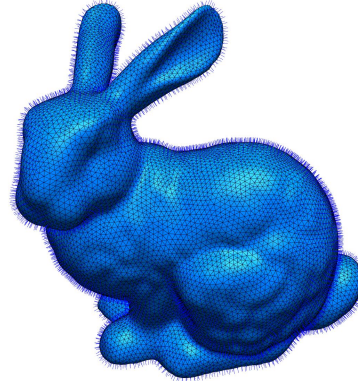
**What's an appropriate data structure
represent polygon-based
surface geometry?**

Mesh Data Structure: Critical Information

- Position (x, y, z)

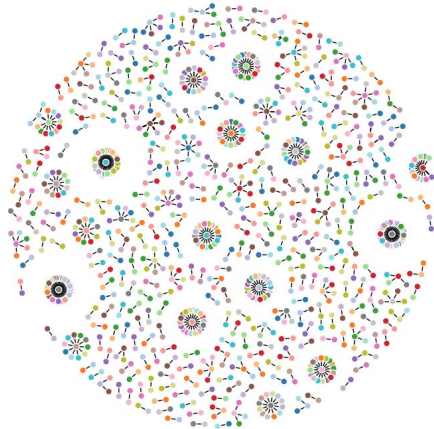


v.s.

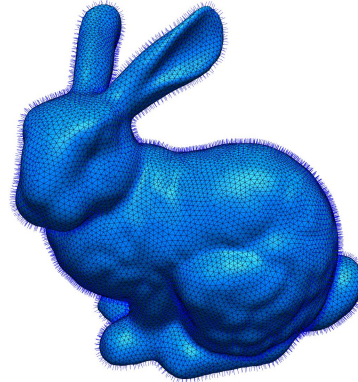


Mesh Data Structure: Critical Information

- Position (x, y, z)
- *Attributes, e.g. per-vertex/face normals, UV coordinates, per-vertex/face colors*

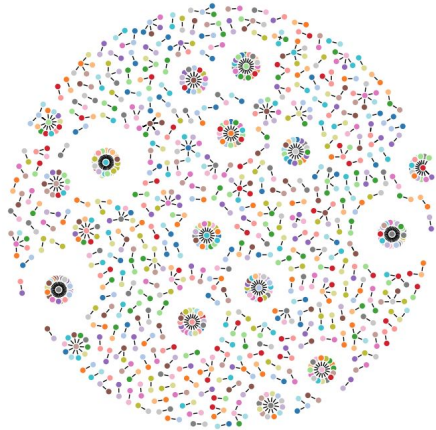


v.s.

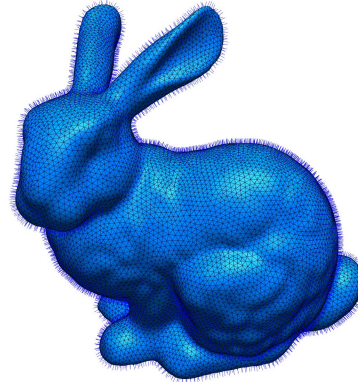


Mesh Data Structure: Geometric Information

- Position (x, y, z)
- *Attributes, e.g. per-vertex/face normals, UV coordinates, per-vertex/face colors*
- **Connectivity**



v.s.

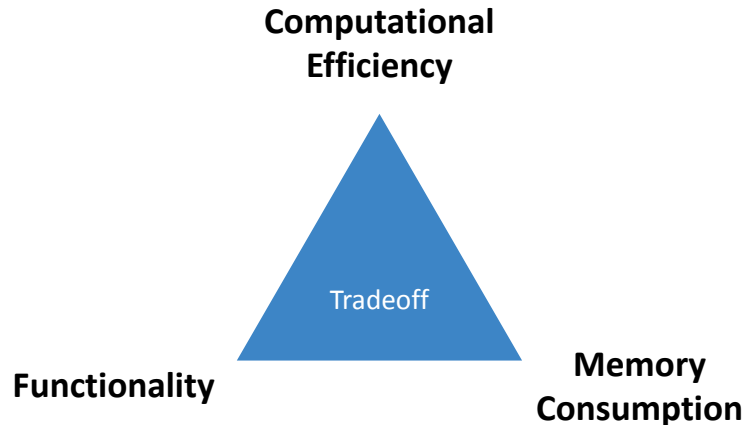


Mesh Data Structure: Optimized on-demand

- Optimize for storage, e.g. persistent on a cache/disk, compressed for consumption
- Optimize for runtime rendering tasks, e.g. Vertex buffer, BVH, fetch efficiency
- Optimize for queries, e.g. What are the vertices and faces of a given face?
- Optimize for manipulation, e.g. Add, remove, collapse, reconstruct a edge/face?
- ...

Mesh Data Structure: Evaluation Criteria

- Preprocessing time, e.g. Time complexity to build a structure from another format
- Query time, e.g. Time complexity of search adjacency faces of a given edge
- Operation time, e.g. Time complexity of removing an edge/face
- Space complexity, e.g. Storage consumption of a structure



Why connectivity is important and how to represent it?

- Connectivity conveys the understanding of *local* information of a vertex
- With connectivity, one can avoid expensive searches
- Different types of connectivity
 - No connectivity: Face set
 - Vertex-based connectivity: Shared vertex
 - Face-based connectivity: Shared face
 - Edge-based connectivity: Shared edge
 - Halfedge-based connectivity

Why connectivity is important and how to represent it?

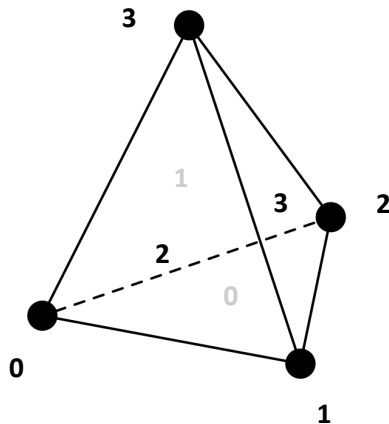
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 - Edge-based connectivity: Shared edge
 - Halfedge-based connectivity
- We will revisit more about why local operations are so important later

Shared Vertex (.obj, .off formats)

Basic Idea: isolate vertex positions, store an *adjacency list* for vertices

- Storage cost
 - 1 floating number = 4 bytes
 - 1 vertex = $4 * 3 = 12$ bytes
 - 1 face = $4 * 2 = 12$ bytes
 - total: $\#v * 12 + \#f * 12 \approx \#v * 12 * 3 = 36$ bytes/vertex
- Pros
 - Still simple, and with small redundancy
- Cons
 - No access to neighbors (why?)

Vertices	Triangles
x1,y1,z1	i11,i12,i13
x2,y2,z3	i21,i22,i23
...	...
xv,yv,zv	...
	...
	in1,in2,in3



Adjacency list for vertices:

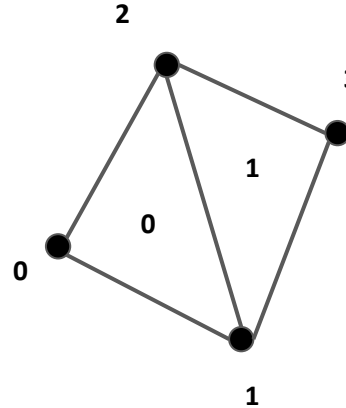
```
0: 0 2 1
1: 0 3 2
2: 3 0 1
3: 3 1 2
```

Face Set (.stl format)

Basic Idea: each row stores the vertices of the face (*array*)

- Storage cost
 - 1 floating number = 4 bytes
 - 1 face = $4 \cdot 9 = 36$ bytes
 - 1 vertex $\approx 2 \cdot 36 = 72$ bytes (why?)
 - total: 72 bytes/vertex
- Pros
 - Very simple
- Cons
 - No connectivity
 - Redundancy (why?)

Triangles		
x11,y11,z11	x12,y12,z12	x13,y13,z13
x21,y21,z21	x22,y22,z22	x23,y23,z23
...
xn1,yn1,zn1	xn2,yn2,zn2	xn3,yn3,zn3



Face array:

x00,y00,z00;x01,y01,z01;x02,y02,z02
x01,y01,z01;x13,y13,z13;x02,y02,z02

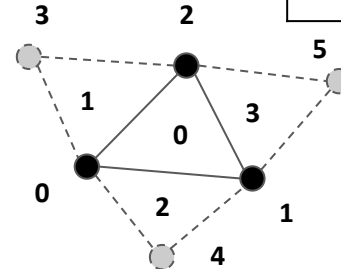
Face-based Connectivity

Basic Idea: store an *adjacency list* for faces and neighboring faces

- Vertex contains
 - Position
 - Associated face
- A face contains
 - Vertices
 - Face neighbors
- Pros
 - Constant time to access all neighboring faces
- Cons
 - No edge information

Vertices	
x_1, y_1, z_1	f1
x_2, y_2, z_2	f2
...	...
x_v, y_v, z_v	f _n

Triangles	
i_{11}, i_{12}, i_{13}	f_{11}, f_{12}, f_{13}
i_{21}, i_{22}, i_{23}	f_{21}, f_{22}, f_{23}
...	
...	
...	
i_{n1}, i_{n2}, i_{n3}	

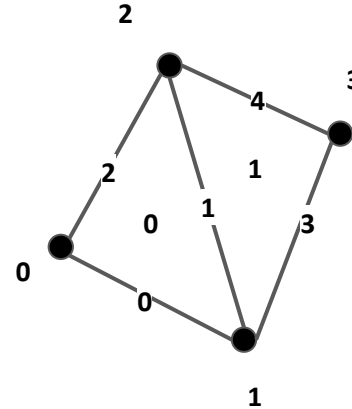


Adjacency List:
 0: 0 1 2; 1 2 3
 1: 0 2 3; 0
 2: 0 4 1; 0
 3: 1 5 2; 0

Edge-based Connectivity

Basic Idea: store an *adjacency list* for edges

- Vertex contains
 - Position
 - 1 adjacent edge index
- A edge contains
 - vertex indices
 - neighboring face indices
 - edges
- A face contains 1 edge index
- Pros: Constant time to access all neighboring faces and edges
- Cons: No edges orientation (why matters?)



Adjacency List:

0: 1 2

1: 0 3 2 4

2: 0 1

3: 4 1

4: 3 1

Incidence Matrix

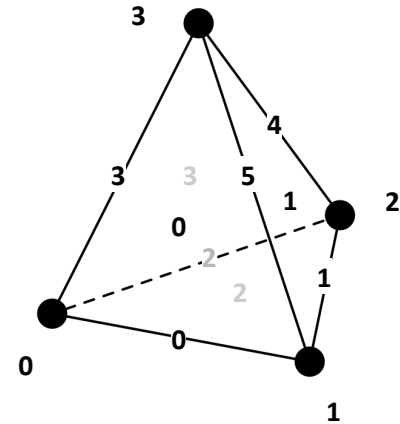
Basic idea: Store **all** neighbor informations via incidence matrices

Vertex-Edge Matrix

	0	1	2	3
0:	1	1	0	0
1:	0	1	1	0
2:	1	0	1	0
3:	1	0	0	1
4:	0	0	1	1
5:	0	1	0	1

Edge-Face Matrix

	0	1	2	3	4	5
0:	1	0	0	1	0	1
1:	0	1	0	0	1	1
2:	1	1	1	0	0	0
3:	0	0	1	1	1	0



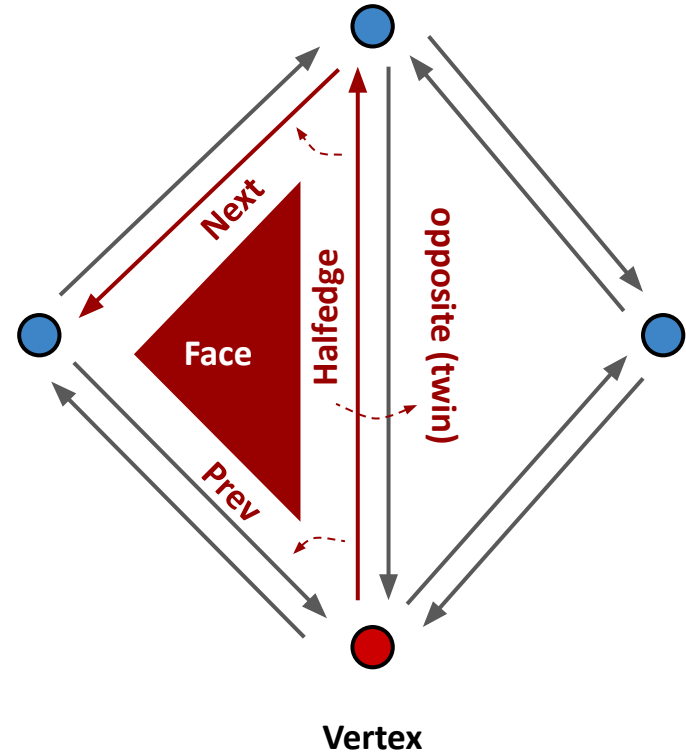
For large meshes, most of the elements will be zero

⇒ Use sparse matrix for tasks at scale

Data Structure: *Halfedge*

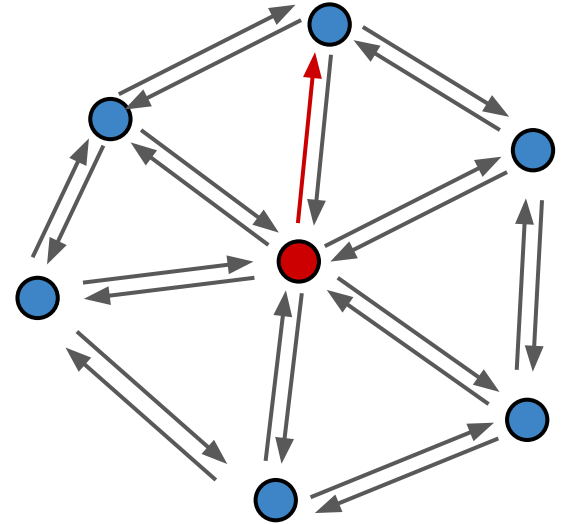
Basic idea: each edge gets split into two half edges

- Vertex
 - Position
 - 1 Halfedge
- Halfedge
 - 1 Vertex
 - 1 Face
 - Prev; Next; Opposite (Twin)
- Face
 - 1 Halfedge



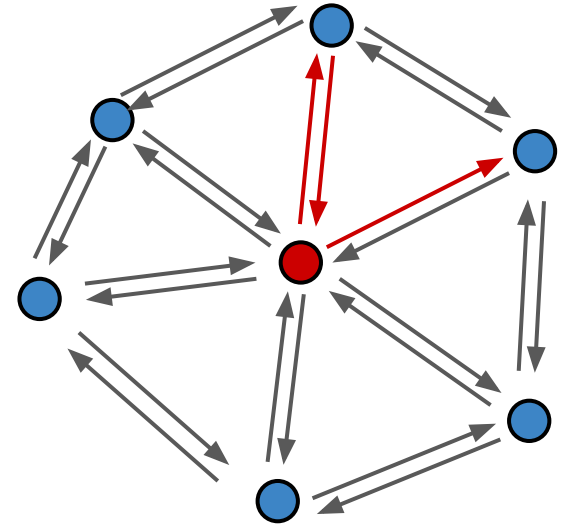
Example 1: Access All Adjacency Edges with Halfedge

```
/**  
 * numAdjacentEdges returns the number of adjacency edges of the given vertex  
 * @param v is an vertex from a halfedge-based mesh  
 */  
function numAdjacentEdges(v: Vertex) {  
    const e0 = v.halfedge  
    let edge_indices = [e0.index]  
    for (let e = e0.opposite.next; e != e0; e = e.opposite.next) {  
        edge_indices.push(e.index)  
    }  
    return edge_indices.length  
}
```



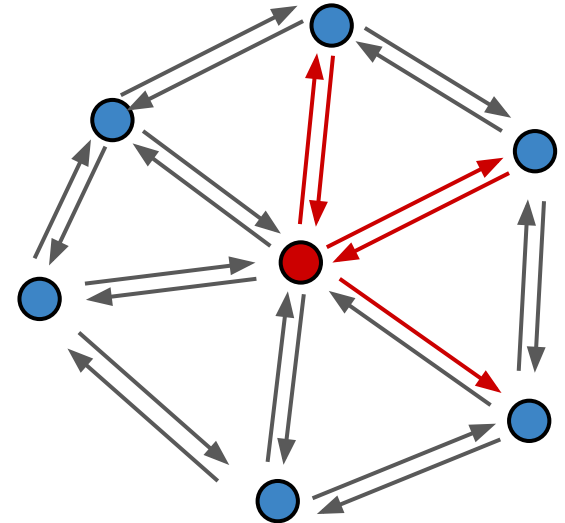
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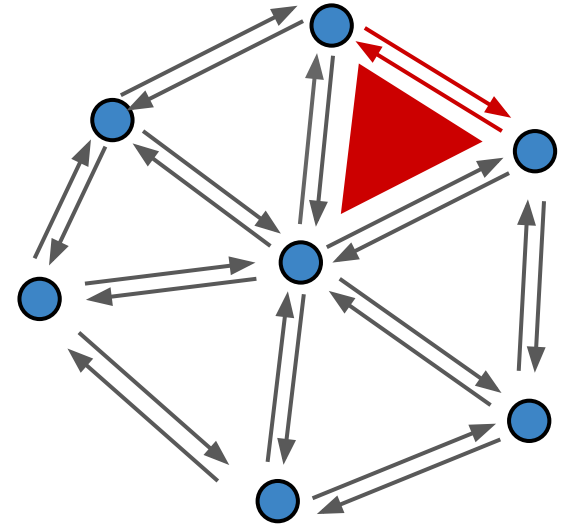
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  }
  return edge_indices.length
}
```



Example 2: Check if an edge is on the boundary of a mesh

```
class Halfedge {  
    ...  
  
    onBoundary() {  
        let connectedFaces = 0  
        if (this.face != null) {  
            connectedFaces++  
        }  
        if (this.opposite.face != null) {  
            connectedFaces++  
        }  
        if (connectedFaces != 1) {  
            return false  
        }  
        return true  
    }  
}
```



See the benefits of halfedge?

- Key benefits: makes traversal easy
 - Easy for editing a mesh: constant time access of local neighbors
- Cons?

Manifold v.s. Non-Manifold

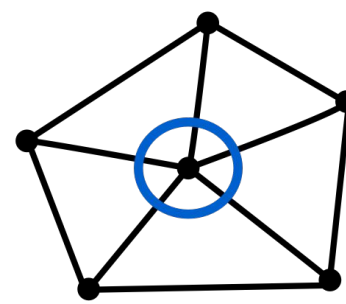
Manifold:

1. Each edge is incident to one or two faces, and
2. faces incident to a vertex from a closed or open fan.

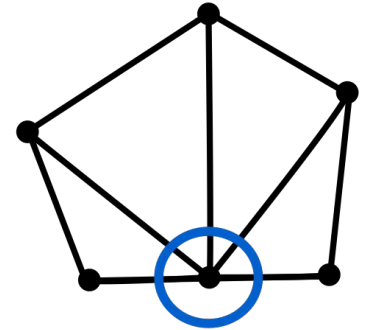
Non-manifold:

1. every edges is constrained in only two polygons (no "fins")
2. the polygons containing each vertex makes a single "fan"

Q: Can halfedge structure deal with non-manifolds?

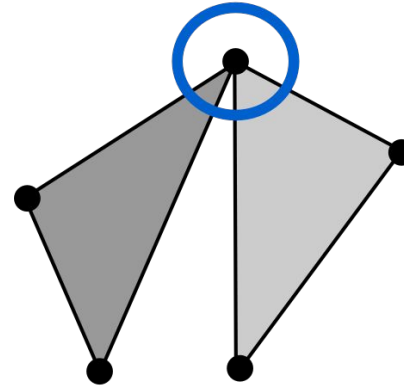


closed "fan"

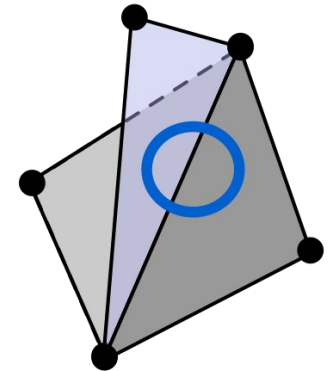


open "fan"

manifolds



on vertex



on edge ("fins")

non-manifolds

Alternatives to Halfedge

Winged edge

Corner table

Quadedge

...

Similar to halfedge and each stores local neighborhood information

Tradeoffs in general:

+ Convenient and better access time for individual elements, easy for traversal of locals

- Additional storage; cache incoherent; ...

More Sophisticated Mesh Data Structures

BMesh from Blender

FbxMesh from Autodesk

FDynamicMesh from Unreal Engine

...

These data structures are not only for geometry processing purpose but also impacts the subsequent workflows, such as animation, rendering, etc.

Why Mesh Representation Is Still an Issue Today?

- Mesh is still the most *efficient/compact/structured* way to represent a solid geometric object
- Mesh structure has a *fundamental impact* on the way we implement an algorithm (why?)

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- Mesh structure has a ***fundamental impact*** on the way we implement an algorithm (why?)
- The increasing interests from the machine learning community, e.g.
 - How to input a mesh to a neural network?
 - How to export the output as a mesh from a neural network?
- Approaching the end of Moore's law: The needs for a concurrent-safe mesh structure

- We will have time to revisit these questions in later sessions.
- Be careful & patient 😊
- Let's now go for some thing maybe more funny...

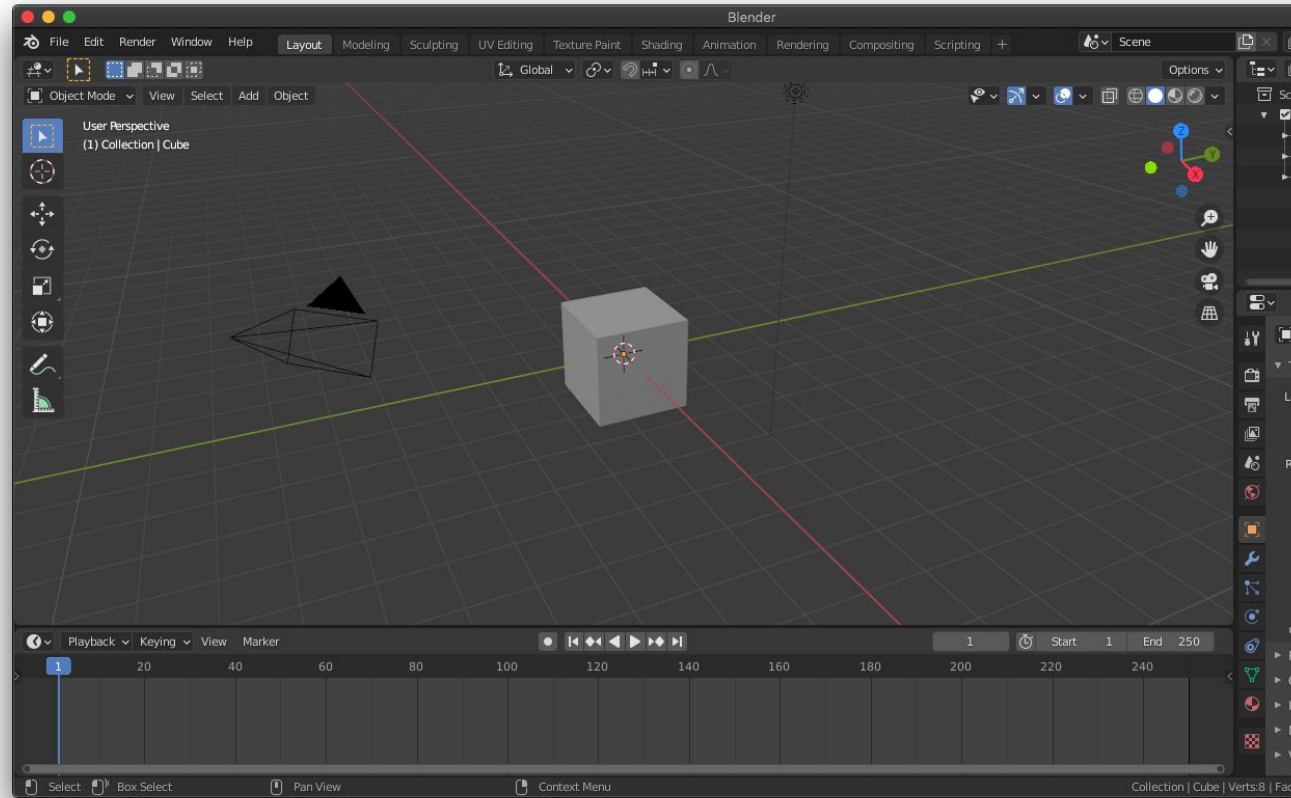
Session 1: Introduction

- Motivation
- Recap: Graphics Pipelines
- Geometry Representations
- Blender Basics
- Summary

Blender

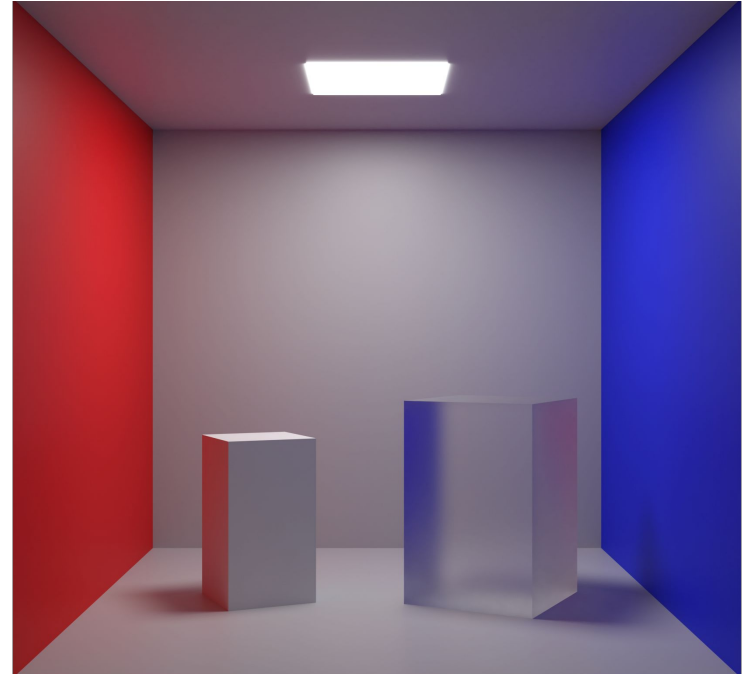
Why not XYZ?

Because of *open source*



Practice: Reproduce The Cornell Box in Blender

- What are the geometry objects do we need for the Cornell box?
- What are their material properties?



Session 1: Introduction

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Summary

- Geometry representation has a long term impact on the geometry processing pipeline
- Mesh is a good compromise and the connectivity is critical for local operations
- No-free Lunch! ⇒ Think about the task, then choose (or design) the right structure
- Understanding the modeling process helps understand more processing workflows

	Adjacency List Based Structure	Incident Matrices Based Structure	Halfedge Based Structure
Constant-time neighborhood access?	No	Yes	Yes
Easy to remove elements?	No	No	Yes
Deal with non-manifold geometry?	Yes	Yes	No